Section D

b) • Reflexivity:

Let (a,b) be an arbitrary element belonging to $N \times N$

So, $(a,b) \in N \times N$.

As, $a - a = b - b = 0$,

$(a,b)\ R\ (a,b)$ for all $(a,b) \in N \times N$

$\therefore$ R is reflexive

• Symmetric:

Let $(a,b), (c,d) \in N \times N$ such that $(a,b)\ R\ (c,d)$.

So,

$a - c = b - d$ [Given Relation]

Multiplying by (-1) on both sides,

$c - a = d - b$

Hence, $(c,d)\ R\ (a,b)$

For $(a,b)\ R\ (c,d)$ there is $(c,d)\ R\ (a,b)$ for $(a,b), (c,d) \in N \times N$

$\therefore$ R is symmetric

- Transitivity:

Let $(a,b), (c,d), (e,f) \in N \times N$ such that $(a,b) R (c,d)$ and $(c,d) R (e,f)$

From the two relations,

$a - c = b - d \quad - (1)$

$c - e = d - f \quad - (2)$

(1) + (2)

$a - c + c - e = b - d + d - f$

So, $a - e = b - f$

Hence, $(a,b) R (e,f)$

For $(a,b) R (c,d)$ and $(c,d) R (e,f) \Rightarrow (a,b) R (e,f)$ for $(a,b), (c$
$(e,f) \in N \times N$

Hence, R is transitive

As R is reflexive, symmetric and transitive, hence R is a
equivalence relation.

As the required line bisects line segment AB, midpoint of line segment AB lies on required line.
Let midpoint of line segment AB be P.

$$P = \left(\frac{2+4}{2}, \frac{3+5}{2}, \frac{4+8}{2} \right) = (3, 4, 6)$$

As required line is perpendicular to two lines,

$$L_1 : \frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } L_2 : \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

the direction vector of required line is found by taking cross product of direction vectors of ' and L_2. Let direction vector of required line by be $\vec{a}$.

$$\vec{a} = (3\hat{i} - 16\hat{j} + 7\hat{k}) \times (3\hat{i} + 8\hat{j} - 5\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$$

$$= \hat{i}(80-56) - \hat{j}(-15-21) + \hat{k}(24+48)$$

$$= 24\hat{i} + 36\hat{j} + 72\hat{k}$$

- DR's of required line = $\langle 24, 36, 72 \rangle = \langle 2, 3, 6 \rangle$
- Simplified Direction vector = $2\hat{i} + 3\hat{j} + 6\hat{k}$

So, Required line is

$$\vec{r} = \vec{OP} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{r} = 3\hat{i} + 4\hat{j} + 6\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$ [Vector equation]

Cartesian equation:

$$\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-6}{6}$$

34)(a) Let $\frac{1}{x} = a$, $\frac{1}{y} = b$, $\frac{1}{z} = c$.

The equations are,

$$2a + 3b + 10c = 4$$
$$4a - 6b + 5c = 1$$
$$6a + 9b - 20c = 2$$

The matrix equation is,

$$\underbrace{\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}}_{\hookrightarrow A} \underbrace{\begin{bmatrix} a \\ b \\ c \end{bmatrix}}_{\hookrightarrow x} = \underbrace{\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}}_{\hookrightarrow B}$$

Its of the form $Ax = B$

$$|A| = 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$$
$$= 2(75) - 3(-110) + 10(72)$$
$$= 150 + 330 + 720$$
$$= 1200$$

As $|A| \neq 0$, above equation have an unique solution

So, $Ax = B$

Pre-multiplying by A^{-1},

$$A^{-1}Ax = A^{-1}B$$
$$Ix = A^{-1}B$$
$$x = A^{-1}B$$

$$A = \begin{bmatrix} \overset{(+)}{2} & \overset{(-)}{3} & \overset{(+)}{10} \\ \overset{(-)}{4} & \overset{(+)}{-6} & \overset{(-)}{5} \\ \overset{(+)}{6} & \overset{(-)}{9} & \overset{(+)}{-20} \end{bmatrix}$$

$$\text{adj}\, A = \begin{bmatrix} 75 & 110 & 72 \\ -(-60-90) & -40-60 & -(18-18) \\ 15+60 & -(10-40) & -12-12 \end{bmatrix}^T$$

$$= \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}^T$$

$$= \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\, \text{adj}\, A$$

$$X = A^{-1}B$$

$$X = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$X = \frac{1}{1200}\begin{bmatrix} 300 + 150 + 150 \\ 440 - 100 + 60 \\ 288 + 0 - 48 \end{bmatrix}$$

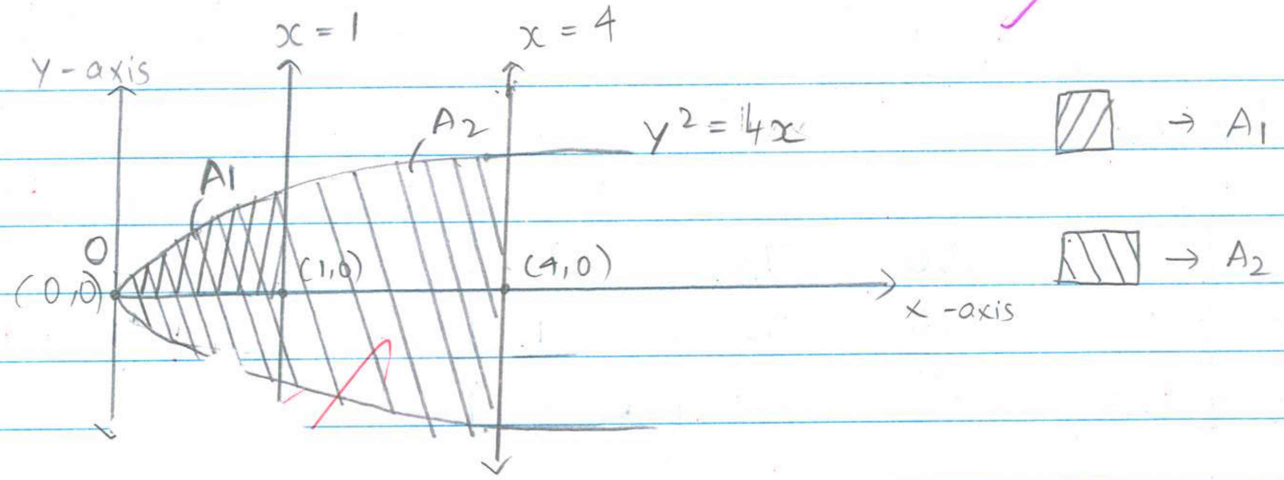
$$X = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$$

So, $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$

Hence, $a = 1/2$, $b = 1/3$, $c = 1/5$

Ans: $x = 2$, $y = 3$, $z = 5$

35)



$y^2 = 4x \Rightarrow y = 2\sqrt{x}$

$A_1 = \int_0^1 y\,dx = 2\int_0^1 \sqrt{x}\,dx = 2\dfrac{(x^{3/2})}{3/2}$

$= \dfrac{4}{3}(x^{3/2})\Big|_0^1 = 4/3\,(1-0) = \dfrac{4}{3}$ sq.units

$A_2 = 2\int_0^4 y\,dx = 2\int_0^4 2\sqrt{x}\,dx = 4\int_0^4 \sqrt{x}\,dx = 4\dfrac{(x^{3/2})}{3/2}$

$= \dfrac{8}{3}(x^{3/2})\Big|_0^4 = \dfrac{8}{3}(4^{3/2}-0) = \dfrac{64}{3}$ sq.units

Ans) $A_1 : A_2 = \dfrac{4}{3} : \dfrac{64}{3} = 1:16$

Section E

)

$$F = \frac{v^2}{500} - \frac{v}{4} + \cancel{\frac{1}{4}}\ 14$$

i) When $v = 40$ km/hr,

$$F = \frac{(40)^2}{500} - \frac{40}{4} + 14$$

$$= \frac{1600}{500} - 10 + 14$$

$$= 3.2 + 4$$

$$= 7.2\ (\ell/100\text{km})$$

Ans: $F = 7.2\ (\ell/100\text{km})$

(ii) $F = \frac{v^2}{500} - \frac{v}{4} + 14$

$\frac{dF}{dV} = \frac{2V}{500} - \frac{1}{4} + 0$

Ans: $\frac{dF}{dV} = \frac{V}{250} - \frac{1}{4}$

iii) (a) $\frac{d^2F}{dV^2} = \frac{1}{250}$

Now, put $\frac{dF}{dV} = 0$

$\frac{V}{250} - \frac{1}{4} = 0 \Rightarrow \frac{V}{250} = \frac{1}{4}$

So, $V = 62.5$ (km/h)

As $\frac{d^2F}{dv^2}$ is positive for $v = 62.5$ km/hr,

F attains minimum at that point.

Ans: $v = 62.5$ km/hr

7)

i) $x + 2y \geq 10$

$x + y \geq 6$

$3x + y \geq 8$

$x \geq 0$

$y \geq 0$

The above are the constraints determining the feasible region ii)

Corner Points	$Z = 16x + 20y$
A (10, 0)	$16 \times 10 + 20 \times 0 = 160$
B (2, 4)	$16 \times 2 + 20 \times 4 = 112$
C (1, 5)	$16 \times 1 + 20 \times 5 = 116$
D (0, 8)	$16 \times 0 + 20 \times 8 = 160$

- The cost is minimum when $x = 2$ and $y = 4$.
- Minimum cost = ₹ 112

38)(i) P(airplane will not crash) $= 1 - \frac{0.00001}{100}$

$= 1 - \frac{10^{-5}}{10^{2}}$

$= 1 - 10^{-7}$

$= 1 - 0.0000001$

$= 0.9999999$

Ans: 0.9999999

i) $P(A/E_1) + P(A/E_2) = 0.95 + 1$

$= 1.95$

ii) a) $P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)$

$= (10^{-7})(0.95) + (1 - 10^{-7})(1)$

$= 0.95 \times 10^{-7} + 1 - 10^{-7}$

$= 1 - 0.05 \times 10^{-7}$

$= 1 - 5 \times 10^{-9}$

$= 1 - 0.000000005$

$= 0.999999995$

Ans: $P(A) = 0.999999995$

Section C

26)(a) $x = e^{\cos 3t}$

Taking log on both sides,

$\log_e x = \cos 3t$ — ①

Differentiating w.r.t t,

$\frac{1}{x}\frac{dx}{dt} = -3\sin 3t$

So, $\frac{dx}{dt} = x(-3\sin 3t)$ — ②

$y = e^{\sin 3t}$

Taking log on both sides,

$\log_e y = \sin 3t$ — ③

Differentiating w.r.t t,

$\frac{1}{y}\frac{dy}{dt} = 3\cos 3t$

$$\frac{dy}{dt} = y\,(3\cos 3t) \quad —④$$

④ ÷ ②

$$\frac{dy}{dx} = \frac{y\,(3\cos 3t)}{x\,(-3\sin 3t)} = \frac{-y}{x}\,\frac{(\cos 3t)}{(\sin 3t)}$$

As $\log x = \cos 3t$ and $\log y = \sin 3t$ from ① and ②,

$$\frac{dy}{dx} = \frac{-y}{x}\,\frac{(\log x)}{(\log y)} = \frac{-y\log x}{x\log y}$$

∴ Hence proved that

$$\frac{dy}{dx} = \frac{-y\log x}{x\log y}$$

27) $I = \int_{-2}^{2} \sqrt{\frac{2-x}{2+x}}\, dx$

Take $x = 2\cos\theta$

$dx = -2\sin\theta\, d\theta$

The limits change

x	2	-2
θ	0	π.

$\begin{bmatrix} 2 = 2\cos\theta \rightarrow \theta = 0 \\ -2 = 2\cos\theta \rightarrow \theta = \pi \end{bmatrix}$

So,

$I = \int_{\pi}^{0} \sqrt{\frac{2-2\cos\theta}{2+2\cos\theta}}\, (-2\sin\theta\, d\theta)$

$I = \int_{0}^{\pi} \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\, (-2\sin\theta\, d\theta)$

$$I = \int_{\pi}^{0} \sqrt{\frac{\cancel{2}\sin^2\theta/2}{\cancel{2}\cos^2\theta/2}} \cdot (-2\sin\theta\, d\theta)$$

$$I = \int_{\pi}^{0} \tan\theta/_2 \,(-2\sin\theta\, d\theta)$$

Using property $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$$I = \int_0^{\pi} (\tan\theta/_2)\,(2\sin\theta\, d\theta) \quad \left[\sin\theta = 2\sin\theta/_2 \cos\theta/_2\right]$$

$$I = \int_0^{\pi} \frac{\sin\theta/_2}{\cancel{\cos\theta/_2}} \,(4\sin\theta/_2 \,\cancel{\cos\theta/_2}\, d\theta)$$

$$I = 4\int_0^{\pi} \sin^2\theta/_2 \, d\theta$$

$$I = 4\int_0^{\pi} \left(\frac{1-\cos\theta}{2}\right) d\theta$$

$$I = 2\int_0^{\pi} (1-\cos\theta)\, d\theta$$

$$= 2\,(\theta - \sin\theta)\Big|_0^{\pi}$$

$$= 2\left((\pi - \sin\pi) - (0 - \sin 0)\right)$$

$$= 2\pi$$

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28)(a) $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0.$

$$\frac{dy}{dx} = \frac{2xy + y^2}{2x^2}.$$

$$\frac{dy}{dx} = \frac{y}{x} + \frac{1}{2}\frac{y^2}{x^2} \quad -①$$

Take $y = vx$.

Differentiating w.r.t x, $\frac{dy}{dx} = v + x\frac{dv}{dx} \quad -②$

Sub $\frac{y}{x} = v$ in (1) and equate (1) and (2)

$$v + x\frac{dv}{dx} = v + \frac{1}{2}v^2$$

$$\frac{x\,dv}{dx} = \frac{v^2}{2}$$

So, on re-arranging,

$$\int \frac{dx}{x} = 2\int \frac{dv}{v^2}$$

Integrating on both sides,

$$\ln|x| + c = 2\int v^{-2}\,dv$$

$$\ln|x| + c = 2\left[\frac{v^{-2+1}}{-2+1}\right] = \frac{-2}{v}$$

$$\ln|x| + c = \frac{-2}{v} = -\frac{2x}{y} \qquad \left[v = \frac{y}{x} \right]$$

So, $c = -\frac{2x}{y} - \ln|x|$

Given: $y(1) = 2$

So, $c = -\frac{2(1)}{2} - \ln(1)$

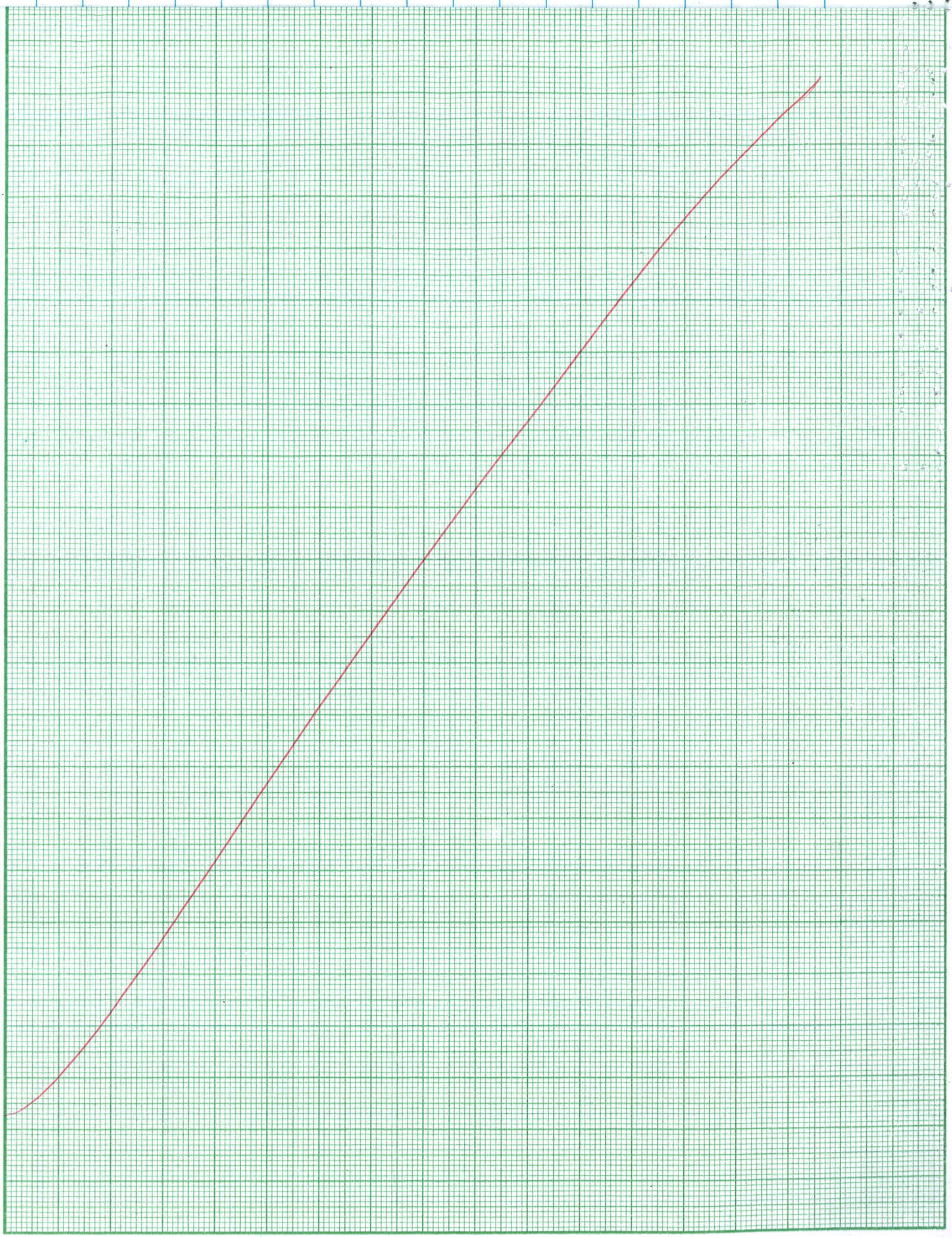
$c = -1$

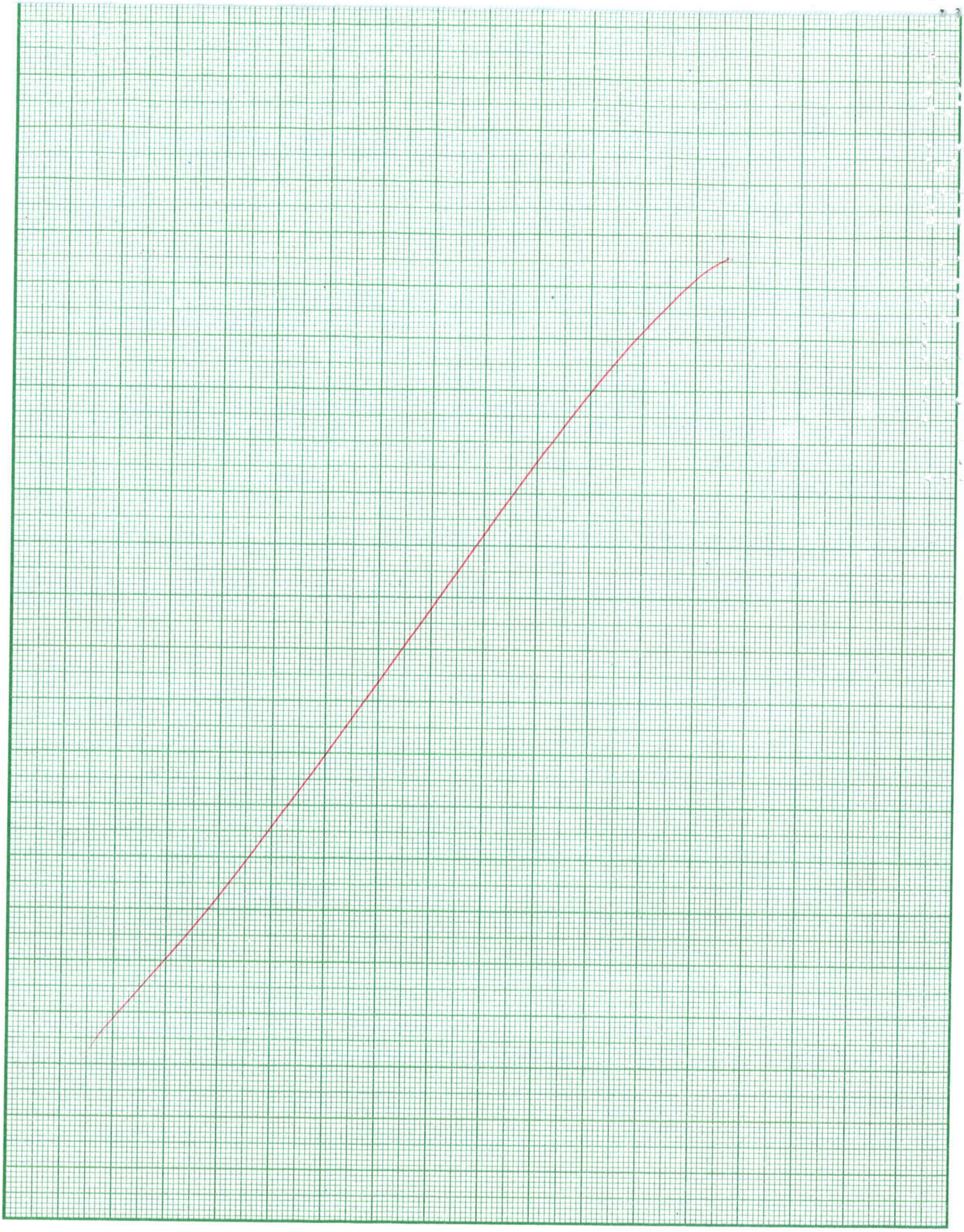
Particular solution is,

$$\ln|x| - 1 = -\frac{2x}{y}$$

$$y = \frac{-2x}{\ln|x| - 1} = \frac{2x}{1 - \ln|x|}$$

Ans: $y = \frac{2x}{1 - \ln|x|}$



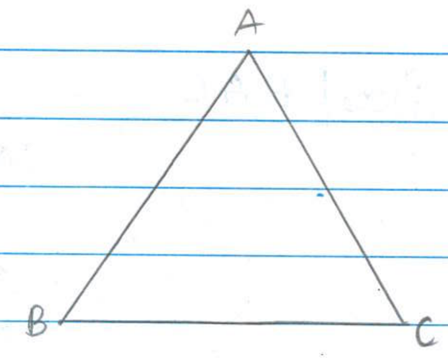


9)

$\vec{OA} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$

$\vec{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$



- $\vec{AB} = \vec{OB} - \vec{OA}$
  $= -\hat{i} - 2\hat{j} - 6\hat{k}$

- $\vec{BC} = \vec{OC} - \vec{OB}$
  $= 2\hat{i} - \hat{j} + \hat{k}$

- $\vec{CA} = \vec{OA} - \vec{OC}$
  $= -\hat{i} + 3\hat{j} + 5\hat{k}$

(i) $\cos \angle ABC = \dfrac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}||\vec{BC}|} = \dfrac{(\hat{i}+2\hat{j}+6\hat{k}) \cdot (2\hat{i} - \hat{j} + \hat{k})}{(\sqrt{1^2+2^2+6^2})(\sqrt{2^2+1^2+1^2}}$

$= \dfrac{2-2+6}{(\sqrt{41})(\sqrt{6})} = \sqrt{\dfrac{6}{41}}$

So, $\angle ABC = \cos^{-1}(\sqrt{6/41})$

(ii) $\cos \angle BAC = \dfrac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}||\vec{AC}|}$

$$= \frac{(-\hat{i} - 2\hat{j} - 6\hat{k}) \cdot (\hat{i} - 3\hat{j} - 5\hat{k})}{(\sqrt{1^2 + 2^2 + 6^2})(\sqrt{1^2 + 3^2 + 5^2})}$$

$$= \frac{-1 + 6 + 30}{(\sqrt{41})(\sqrt{35})} = \sqrt{\frac{35}{41}}$$

$$\angle BAC = \cos^{-1}\left(\sqrt{\frac{35}{41}}\right)$$

(iii) $\cos \angle ACB = \dfrac{\vec{CA} \cdot \vec{CB}}{|\vec{CA}||\vec{CB}|}$

$$= \frac{(-\hat{i} + 3\hat{j} + 5\hat{k}) \cdot (-2\hat{i} + \hat{j} - \hat{k})}{(\sqrt{1^2 + 3^2 + 5^2})(\sqrt{2^2 + 1^2 + 1^2})} = \frac{2 + 3 - 5}{(\sqrt{35})(\sqrt{6})}$$

So, $\angle ACB = \pi/2$

Ans: $\angle BAC = \cos^{-1}\sqrt{\frac{35}{41}}$, $\angle ABC = \cos^{-1}\sqrt{\frac{6}{41}}$, $\angle ACB = \pi/2$

30) Let x denote the absolute difference of numbers appearing on top of dice

x can take values 0, 1, 2, 3, 4, 5.

- For $x = 0 \rightarrow \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

  ↳ $P(x = 0) = \frac{6}{36} = \frac{1}{6}$

- For $x = 1 \rightarrow \{(1,2), (2,3), (3,4), (4,5), (5,6), (2,1), (3,2), (4,3), (5,4), (6,5)\}$

  ↳ $P(x = 1) = \frac{10}{36} = \frac{5}{18}$

- For $x = 2 \rightarrow \{(1,3), (2,4), (2,5), (4,6), (3,1), (4,2), (5,3), (6,4)\}$

  ↳ $P(x = 2) = \frac{8}{36} = \frac{2}{9}$

- For $x = 3 \rightarrow \{(1,4), (2,5), (3,6), (4,1), (5,2), (6,3)\}$

  ↳ $P(x = 3) = \frac{6}{36} = \frac{1}{6}$

- For $x = 4 \rightarrow \{(1,5), (2,6), (5,1), (6,2)\}$

  ↳ $P(x = 4) = \frac{4}{36} = \frac{1}{9}$

- For $x = 5 \rightarrow \{(1,6), (6,1)\}$

  ↳ $P(x = 5) = \frac{2}{36} = \frac{1}{18}$

The Probability distribution table is,

x	0	1	2	3	4	5
$P(x)$	$1/6 = 6/36$	$10/36 = 5/18$	$8/36 = 2/9$	$6/36 = 1/6$	$4/36 = 1/9$	$2/36 = 1/18$

31) $I = \int x^2 \cdot \sin^{-1}(x^{3/2})\, dx$

Take $t = x^3$

$dt = 3x^2 dx \rightarrow \frac{dt}{3} = x^2 dx$

So, $I = \frac{1}{3}\int (1)\sin^{-1}(\sqrt{x})\, dx$

$\sqrt{x} = t$

$\frac{1}{2\sqrt{x}} dx = dt \quad dx = 2t\,dt$

$\frac{1}{3}\int 2t \sin^{-1} t\, dt$

$\frac{1}{3}\; t^2 \sin^{-1} t - \int \frac{t^2}{\sqrt{1-t^2}}\, dt$

Take $U = \sin^{-1}(\sqrt{x})$, $dV = dx$

Applying By - Parts,

$3I = x\sin^{-1}x - \int x\left(\frac{1}{\sqrt{1-x}}\right) \cdot \left(\frac{1}{2\sqrt{x}}\right) dx$

$$3I = x\sin^{-1}x - \frac{1}{2}\int \sqrt{\frac{x}{1-x}}\,dx$$

Take $\int \sqrt{\frac{x}{1-x}}\,dx = I_1$

Put $1-x = t^2 \rightarrow x = 1-t^2$

$-dx = \cancel{2t}\,dt \rightarrow -\overset{2t}{}dt = +dx$

$$I_1 = -\int \sqrt{\frac{1-t^2}{t}}\,(dt)(2t)$$

$$= -2\int \sqrt{t(1-t^2)}\,dt$$

Section - B

21) (a) $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}(-1)$

$= -\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \pi/_3 - \tan^{-1}(1)$

$= -\pi/_6 + \pi/_3 - \pi/_4$

$= \pi/_3 - \frac{5\pi}{12} = -\frac{\pi}{12}$

Ans: $\frac{-\pi}{12}$

)b) $y = \operatorname{cosec}(\cot^{-1}x)$

$\frac{dy}{dx} = -\operatorname{cosec}(\cot^{-1}x).\cot(\cot^{-1}x).\left(\frac{-1}{\sqrt{1+}}\right.$

$= -x\operatorname{cosec}(\cot^{-1}x)$ $\quad [\operatorname{cosec}\theta = \sqrt{1+\cot^2\theta}]$

$= -x\sqrt{1+\cot^2(\cot^{-1}x)}$

$= -x\sqrt{1+x^2}$

2)b) $y = \operatorname{cosec}(\cot^{-1}x)$

Diff w.r.t x,

$\frac{dy}{dx} = -\operatorname{cosec}(\cot^{-1}x).\cot(\cot^{-1}x).\left(\frac{-1}{1+x^2}\right)$ $\quad [\cot(\cot^{-1}x) = x]$

$\frac{dy}{dx} = \frac{x\operatorname{cosec}(\cot^{-1}x)}{1+x^2}$ $\quad [\operatorname{cosec}\theta = \sqrt{1+\cot^2\theta}]$

$$\frac{dy}{dx} = \frac{x\sqrt{1+\cot^2(\cot^{-1}x)}}{1+x^2}$$

$$\frac{dy}{dx} = \frac{x(\sqrt{1+x^2})}{1+x^2}$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}}$$

So, $(\sqrt{1+x^2})\frac{dy}{dx} = x$

$$(\sqrt{1+x^2})\frac{dy}{dx} - x = 0.$$

Hence, Proved the required

3) $f(x) = x + \frac{1}{x}$

Diff wrt x on both sides

$f'(x) = 1 - \frac{1}{x^2}$

Put $f'(x) = 0$

$1 - \frac{1}{x^2} = 0 \Rightarrow 1 = \frac{1}{x^2} \Rightarrow x^2 = 1$

So, $x = \pm 1$.

Now, $f''(x) = \frac{2}{x^3}$

$f''(1) = \frac{2}{1} > 0$.

$f(x)$ attains minima at $x = 1$.

- $f''(-1) = \frac{2}{-1} < 0$

  $f(x)$ attains maxima at $x = -1$.

So, $\cancel{M = -1}$, $\cancel{m = 1}$

$\cancel{M - m = -1 -}$

- Local maximum value $= M = f(-1) = -1 + \frac{1}{(-1)} = -2$

- Local minimum value $= m = f(1) = 1 + \frac{1}{1} = 2$

So, $M - m = -2 - (2)$

$= -4$

Ans: -4

24) $I = \int \frac{(e^{4x}-1)}{e^{4x}+1}\,dx$

Divide Numerator and Denominator ~~by~~ by e^{2x}

So,

$I = \int \frac{(e^{2x}-e^{-2x})}{e^{2x}+e^{-2x}}\,dx$

Take $U = e^{2x} + e^{-2x}$

$dU = (2e^{2x} - 2e^{-2x})\,dx$

So, $\frac{dU}{2} = (e^{2x} - e^{-2x})\,dx$

Hence, $I = \frac{1}{2}\int \frac{dU}{U} = \frac{1}{2}\log_e |U| + c$

Ans: $I = \frac{1}{2}\log_e |e^{2x} + e^{-2x}| + c$

25) $f(x) = e^{+x} - e^{-x} + x - \tan^{-1}x$

Differentiating w.r.t x,

$$f'(x) = e^{x} + e^{-x} + 1 - \frac{1}{1+x^2}$$

$$= e^{x} + e^{-x} + \frac{1+x^2-1}{1+x^2}$$

$$= e^{x} + e^{-x} + \frac{x^2}{1+x^2}$$

For any value of x in its domain,

$e^{x} > 0$, $e^{-x} > 0$, $\frac{x^2}{1+x^2} > 0$ $\left[\frac{-1}{1+x^2} \geq -1 \rightarrow 1 - \frac{1}{1+x^2} \right.$

So, $f'(x) = e^{x} + e^{-x} + \frac{x^2}{1+x^2} > 0$

As $f'(x)$ is always greater than 0, $f(x)$ is a strictly increasing function

Section - A

) Product = 0

Ans: (A) 0

?) $f(x) = 9x^2 + 6x - 5$

$= (3x)^2 + 2(3x)(1) + 1 - 6$

~~$(3x+1)^2 - 6$~~

$= (3x+1)^2 - 6$

C Given: $x \geq 0$

$3x \geq 0$

$3x + 1 \geq 1 \rightarrow (3x+1)^2 \geq 1$

So, $(3x+1)^2 - 6 \geq -5$

Ans: (c) Bijective

3) $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = abc \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$= abc\left[-1(1-1) - 1(-1-1) + 1(1+1)\right]$

$= abc\,[2+2]$

$= 4abc = kabc \quad [k = 4]$

Ans: (D) 4

4) At $x = -3$:

- $LHL = |-3| + 3 = 6 = F(-3)$
- $RHL = -2(-3) = 6$

At $x = 3$:

- $LHL = -2(3) = -6$
- $RHL = F(3) = 6(3) + 2 = 20$

Ans: (B) 1

) $f(x) = x^3 - 3x^2 + 12x - 18$

$f'(x) = 3x^2 - 6x + 12$

$= 3(x^2 - 2x + 4)$

$= 3(x-2)^2$

Ans: (B) strictly increasing on R.

6) $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx = \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx$

Ans: (B) Zero(0)

7) (A) $\cos x - \sin\left(\frac{y}{x}\right)$

8) (C) $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$

1) (D) (0, 0, 0)

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10, (D) a feasible region

11) $P(S/E) = \frac{P(S \cap E)}{P(E)} = \frac{P(E)}{P(E)} = 1$

Ans: (C) 1

12) • $a_{11} = 1 - 3 = -2$

• $a_{12} = 1 - 6 = -5$, $a_{21} = 2 - 3 = -1$

So, $a_{12} + a_{21} = -6$

• $a_{13} = 1 - 9 = -8$, $a_{31} = 3 - 3 = 0$

Ans: (C) $a_{13} > a_{31}$

13) $\frac{d(\tan^{-1}(x^2))}{dx} = \frac{1}{1+x^4}(2x) = \frac{2x}{1+x^4}$

Ans: (B) $\frac{2x}{1+x^4}$

1) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = x\sin\left(\frac{dy}{dx}\right)$

Ans: (D) not defined

i) $(\hat{i}+\hat{k}) \times (\hat{i}-\hat{k}) = \hat{j} + \hat{j} = 2\hat{j}$ [Unit vector $= \hat{j}$]

Ans: (B) $\hat{j}$

i) $\frac{x-1}{2} = \frac{y}{-1} = \frac{z+1/2}{3}$

Ans: (D) 2, −1, 3

1) $[F(x)]^2 = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos 2x & -\sin 2x & 0 \\ \sin 2x & \cos 2x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Ans: (B) 2

18) $\alpha = 30^\circ, \ \beta = 120^\circ$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2 30 + \cos^2 120 + \cos^2\gamma = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{-1}{2}\right)^2 + \cos^2\gamma$

$= 1 + \cos^2\alpha$

So, $\cos\gamma = 0 \Rightarrow \gamma = 90^\circ$

Ans: (A) 90°

19) $(B'AB)^T = B^T A^T B$

$= B'AB \quad [A = A']$

Ans: (D) Assertion (A) is False, but Reason (R) is true

20) (C) Assertion (A) is true, but Reason (R) is false.

$$I = \int x^2 \sin^{-1}(x^{3/2})\,dx$$

Let $t^2 = x^3 \rightarrow \frac{2t\,dt}{3} = x^2 dx$

So,

$$\cancel{I = \frac{1}{3}\int \sin^{-1}(\sqrt{t})\,dt}$$

$$I = \frac{1}{3}\int 2t \sin^{-1}t\,dt$$

Now, $dv = 2t\,dt$ and $u = \sin^{-1}t$

Applying By-parts $[uv - \int v\,du]$,

$$3I = t^2\sin^{-1}t - \int \frac{t^2}{\sqrt{1-t^2}}\,dt$$

$$= t^2\sin^{-1}t + \int \frac{(1-t^2-1)}{\sqrt{1-t^2}}\,dt$$

$$= t^2\sin^{-1}t + \int (\sqrt{1-t^2})\,dt - \int \frac{1}{\sqrt{1-t^2}}\,dt$$

$$3I = t^2 \sin^{-1} t + \frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1}(t) - \sin^{-1} t + C$$

$$= \left(t^2 - \tfrac{1}{2}\right)\sin^{-1} t + \frac{t}{2}\sqrt{1-t^2} + C$$

Subbing $t^2 = x^3$,

$$3I = \left(x^3 - \frac{1}{2}\right)\sin^{-1}(x^{3/2}) + \frac{x^{3/2}}{2}\sqrt{1-x^3} + C$$

So,

$$I = \frac{1}{3}\left[\left(x^3 - \frac{1}{2}\right)\sin^{-1}(x^{3/2}) + \frac{x^{3/2}}{2}\sqrt{1-x^3}\right] + c' \quad \left[c' = c/3\right]$$

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