Roll Number		

SET A

INDIAN SCHOOL MUSCAT
HALF YEARLY EXAMINATION
SUBJECT : MATHEMATICS

CLASS: XII | Sub.Code:041 | Time Allotted: 3 Hrs.

22.09.2019 | Max.Marks: 80

General Instructions:

(i) All questions are compulsory.
(ii) This question paper contains 36 questions.
(iii) Question 1- 20 in Section A are MCQ/Very short-answer type questions carrying 1 mark each.
(iv) Question21-26 in Section B are short-answer type questions carrying 2 marks each.
(v) Question 27-32 in Section C are long-answer-I type questions carrying 4 marks each.
(vi) Question 33-36 in Section D are long-answer-II type questions carrying 6 marks each.

	SECTION A	
1.	If f,g : R→ R be two functions defined as f(x) = $\|x\| + x$ and g(x) = $\|x\| - x$,for all x in R,find fog(-5).	1
2.	Find the value of $\cos^{-1}\cos\left(\frac{7\pi}{6}\right)$.	1
3.	Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$	1
4.	Find the area bounded by the curve y = cosx, between x = 0 and x = 2π .	1
5.	Evaluate: $\int logx\,dx$	1
6	Evaluate: $\int_{-1}^{1} [x]dx$	1
7.	Evaluate : $\int_0^{2\pi} sinx\,dx$	1
8.	Evaluate: $\int \frac{1-cos2x}{1+cos2x}\,dx$	1
9.	Find the area bounded by the lines y = x and x = 1 in the first quadrant.	1
10.	A point C in the domain of a function f at which either $f'(c) = 0$ or f is not differentiable is called --------------- .	1

11.	f(x) = $\begin{cases} ax^2+1\ ,\ x>1 \\ x+a\ , x\le 1 \end{cases}$ is differentiable at x = 1, then find the value of a. a) 2 b) 1 c) 0 d) $\frac{1}{2}$	1
12.	f(x) = $\begin{cases} xSin\ \frac{1}{x}, x\neq 0 \\ k\ ,\ x=0 \end{cases}$ is continuous at x = 0. Find k. a) 8 b) 1 c) -1 d) 0	1
13.	If $y = x + e^x$, then $\frac{d^2x}{dy^2}$ = ------------- $a)$ $\frac{1}{(1+e^x)^2}$ b) $\frac{-e^x}{(1+e^x)^2}$ c) $\frac{-e^x}{(1+e^x)^3}$ d) e^x	1
14.	Let R be the relation in the set N given by R = $\{(a,b): a = b-2, b>6\}$.Choose the correct answer. A) (2,4) ∈ R B) (3,8) ∈ R C) (6,8) ∈ R D) (8,7)∈ R	1
15.	Let f : R→ R be defined as f(x) = x^4.Choose the correct answer. a)F is one- one onto b) f is many-one onto c) f is one-one but not onto d) f is neither one-one nor onto.	1
16.	The interval in which $y = x^2e^{-x}$ is increasing is $a)$ $(-\infty,\infty)$ b) $(-2,0)$ c) $(2,\infty)$ d) $(0,2)$	1
17.	The line y = x + 1 is a tangent to the curve y^2 = 4x at the point a) (1, 2) b) (2 , 1) c) (1, -2) d) (-1, 2)	1
18.	Choose the correct principal value branch of the range of $y = \tan^{-1} x$. $a)$ $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ b) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ c) $[0,\pi]$ d) $(0,\pi)$	1
19.	Find the area bounded by f(x) = $\vert x\vert$, between x = -3 and x = 3. a) 0 b) 18 sq.units c) 9 sq.units d) 6 sq.units	1
20.	Find the derivative of Sin$(x)^3$with respect to Cos$(x)^3$. $a) -tan(x^3)$ b) -$cot(x^3)$ c) $cot(x^3)$ d) $tan(x^3)$	1
	SECTION B	
21.	Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{2}{11}\right) = \tan^{-1}\left(\frac{3}{4}\right)$ OR Evaluate: $sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$	2
22.	Find the value of k ,if the following function is continuous at 1 f(x) = $\begin{cases} k(x^2-2)\ ,\ x \le 1 \\ 4x+1\ ,\ x>1 \end{cases}$	2
23.	Find $\frac{dy}{dx}$ if, y = $\sin^{-1}(\frac{1-x^2}{1+x^2})$ 0<x<1	2

24.	Find $\int_1^4 f(x)dx\,,\ if(x) = \begin{cases} 7x\ ;if\ 1 \le x \le 3 \\ 8\ \ ;\ if\ : 3 \le x\ \le 4 \end{cases}$ OR $.Evaluate: \int \frac{5^{(7x-5)}}{5^{(2x+10)}}\ dx$	2
25.	The total cost c(x) associated with the production of x units of an item is given by C(x) = $0.007x^3$-$0.003x^2$+15x +4000.Find the marginal cost when 17units are produced.	2
26.	Evaluate: $\int \sqrt{\frac{a+x}{a-x}} - \sqrt{\frac{a-x}{a+x}}$ dx	2
	SECTION C	
27.	Simplify : $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$	4
28.	*f* : **N** →**N** be defined by f(x) = $\begin{cases} x+1, if\ x\ is\ odd \\ x-1, if\ x\ is\ even \end{cases}$ for all $x \epsilon$ N, show that f is bijective.	4
29.	Find the intervals in which the functions given below are strictly decreasing or strictly increasing:- f(x) = $\frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}$ x +11 OR Find the equations of the tangent and normal to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point ,where it cuts x-axis.	4
30.	Find $\frac{dy}{dx}$, y = $(sinx)^{cosx} + x^{sinx}$	4
31.	If f(x) = $\begin{cases} x^2+3x+a,\ x \le 1 \\ bx+2,\ x > 1 \end{cases}, is\ differentiable. Find\ a\ and\ b.$ OR If f(x) = $\begin{cases} \frac{1-sin^3x}{3cos^2x}, x < \frac{\pi}{2} \\ a\,,\ x = \frac{\pi}{2} \\ \frac{b(1-sinx)}{(\pi-2x)^2}, x > \frac{\pi}{2} \end{cases}$ is continuous at x = $\frac{\pi}{2}$, find a and b.	4
32.	Evaluate: $\int \frac{x+2}{2x^2+6x+5}$ dx	4

	SECTION D	
33.	Let f: **N**→ $\boldsymbol{R}$ be a function defined as f(x) = $4x^2$ +12x +15 . show that f:**N**→ $\boldsymbol{S}$, where **S** is the range of f is invertible .Find the inverse of f. OR Show that the relation R in the set N of Natural numbers given by R = $\{(a, b): \|a - b\|\ is\ a\ multiple\ of\ 3\}$ is an equivalence relation.	6
34.	Find the area of the region enclosed between the two circles $x^2 + y^2 = 4\ and$ $(x-2)^2 + y^2 = 4$ OR Using integration find the area of region bounded by the triangle whose vertices are (1,0),(2,2) and (3,1).	6
35.	Evaluate: $\int \sqrt{\tan x} + \sqrt{\cot x}\ dx$	6
36.	Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.	6