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# TESTS OF DIVISIBILITY

There are certain tests of divisibility that can help us to decide whether a given number is divisible by another number.

**1. Divisibility of numbers by 2:**

► A number that has 0, 2, 4, 6 or 8 in its ones place is divisible by 2.

**2. Divisibility of numbers by 3**

► A number is divisible by 3 if the sum of its digits is divisible by 3.

**3. Divisibility of numbers by 4**

► A number is divisible by 4 if the number formed by its last two digits (i.e. ones and tens) is divisible by 4.

**4. Divisibility of numbers by 5**

► A number that has either 0 or 5 in its ones place is divisible by 5.

**5. Divisibility of numbers by 6:**

► A number is divisible by 6 if that number is divisible by both 2 and 3.

**6. Divisibility of numbers by 7:**

► A number is divisible by 7, if the difference b/w twice the last digit and the no. formed by the other digits is either 0 or a multiple of 7. eg. 2975, it is observed that the last digit of 2975 is '5', so, 297 --(5x2) = 297 - 10 =287, which is a multiple of 7 hence, it is divisible by 7

**7. Divisibility of numbers by 8:**

► A number is divisible by 8 if the number formed by its last three digits is divisible by 8.

**8. Divisibility of numbers by 9:**

► A number is divisible by 9 if the sum of its digits is divisible by 9.

**9. Divisibility of numbers by 10:**

► A number that has 0 in its ones place is divisible by 10.

**10. Divisibility of numbers by 11:**

► If the difference between the sum of the digits at the odd and even places in a given number is either 0 or a multiple of 11, then the given number is divisible by 11.

**11. Divisibility of number by 12.**

► Any number which is divisible by both 4 and 3, is also divisible by 12. To check the divisiblity by 12, we i. First divide the last two-digit number by 4. If it is not divisible by 4, it is divisible by 4 is not divisible by 12. If it is divisible by 4 them. ii. Check whether the number is divisible by 3 or not.

Ex: 135792 : 92 is divisible by 4 and also (1 + 3 + 5 + 7 + 9 +2 =) 27 is divisible by 3 ; hence the number is divisible by 12.

**12. Divisibility by 13**

Oscillator for 13 is 4. But this time, our oscillator is not negative (as in case of 7) It is 'one-more' Oscillator. So, the working Principle will be different now.

**Eg: Is 143 divisible by 13** ? Sol: 14 3 : 14 + 3 x 4 = 26 Since 26 is divisible by 13, the number 143 is also divisible by 13. Eg 2 : Check the divisibility by 13. 2 416 7 26/6/20/34 [ 4 x 7 ( from 24167 ) + 6 ( from 24 167) = 34] [4 x 4 ( from 3 4 ) + 3 (from 3 4 ) + 1 (from 24167)] =20 [4 x 0 (from 2 0 ) + 2 (from 20) + 4 (from 24 167)= 6] [4 x 6 (from 6 ) + 2 (from 24 167)= 26] Since 26 is divisible by 13 the number is also divisible by 13.



**13. Divisibility by 14**

► Any Number which is divisible by both 2 and 7, in also divisible by 14. That is, the number's last digit should be even and at the same time the number should be divisible by 7.

**14. Divisibility by 15**

► Any number which is divisible by both 3 and 5 is also divisible by 15.

**15. Divisibility by 16**

► Any number whose last 4 digit number is divisible by 16 is also divisible by 16.

**16. Divisibility by 17**

► Negative Oscillator for 17 is 5. The working for this is the same as in the case 7. Eg: check the divisibility of 1904 by 17

Sol: 1904 : 190 - 5 x 4 = 170 Since 170 is divisible by 17, the given number is also divisible by 17. E.g 2: 957508 by 17

So1:95750 8: 95750 - 5 x 8 = 95710 9571 0 : 9571 - 5 x 0 = 9571 957 1 : 957 - 5 x 1 = 952 952 : 95 - 5x2 =85

Since 85 is divisible by 17, the given number is divisible by 17.

**17. Divisibility by 18**

► Any number which is a divisible by 9 has its last digit (unit-digit) even or zero, is divisible by 18. Eg. 926568 : Digit - Sum is a multiple of nine (i.e, divisible by 9) and unit digit (8) is even, hence the number is divisible by 18.

**18. Divisibility by 19**

► If recall, the 'one-more' osculator for 19 is 2. The method is similar to that of 13, which is well known to us. Eg. 1 4 9 2 6 4 19/9/12/11/14

**General rules of divisibility for all numbers:**

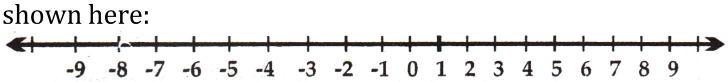
♦ If a number is divisible by another number, then it is also divisible by all the factors of the other number.

♦ If two numbers are divisible by another number, then their sum and difference is also divisible by the other number.

♦ If a number is divisible by two co-prime numbers, then it is also divisible by the product of the two co-prime numbers.

## INTEGERS

• Whole numbers are represented on the number line as shown here:




-9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9

• If you move towards the right from the zero mark on the number line, the value of the numbers increases.

▪ If you move towards the left from the zero mark on the number line, the value of the numbers decreases.

i. Integers: The collection of the numbers, that is, ... -3, -2, -1, 0, 1, 2, 3, ...., is called integers.

ii. Negative integers: The numbers -1, -2, -3, -4... which are called negative numbers.

iii. Positive integers: The number 1, 2, 3, 4 ...s, which are called positive

• Euclid's division lemma can be used to: $a = b \times q + r$

• Find the highest common factor of any two positive integers and to show the common properties of numbers.

• Finding H.C.F using Euclid's division lemma.

• Suppose, we have two positive integers 'a' and 'b' such that 'a' is greater than 'b'. Apply Euclid's division lemma to the given integers 'a' and 'b' to find two whole numbers 'q' and 'r' such that, 'a' is equal to 'b' multiplied by 'q' plus 'r'.

• Check the value of 'r':

If 'r' is equal to zero then 'b' is the **HCF** of the given numbers.

If 'r' is not equal to zero, apply **Euclid's division lemma** to the new divisor 'b' and remainder 'r'. Continue this process till the remainder 'r' becomes zero. The value of the divisor 'b' in that case is the **HCF** of the two given numbers.

• Euclid's division algorithm can also be used to find some common properties of numbers.

**Some Rules on Counting Numbers**

**i. Sum of all the first n natural numbers** = $\frac{n\,(n+1)}{2}$

***For eg.:*** **1 + 2 + 3 +....+ 105** = $\frac{105\,(105+1)}{2}$ =5565

**ii. Sum of first n odd numbers = $n^2$**

**Eg: 1 + 3 + 5 + 7 = $4^2$ = 16 (as there are four odd numbers)**

**Eg: 1 + 3 +5 + +20th odd numbers**

(ie. $20 \times 2 - 1 = 39$ ) = $20^2 = 400$

**iii. Sum, of first n even numbers = n(n + 1)**

**Eg: 2 + 4 +6 +8 + +100 (or 50th Even number)**

= 50 x (50 + 1 )= 2550

**iv. Sum of Squares of first n natural numbers** = $\frac{n\,(n+1)(2n+1)}{6}$

For eg : $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots 10^2$

$= \frac{10\,(10+1)(2\times10+1)}{6} = \frac{10\times11\times21}{6} = 385$

**v. Sum of cubes of first n Natural numbers**

$= \left[\frac{n\,(n+1)}{2}\right]^2$

**For eg : $1^3 + 2^3$ + + ............ $6^3$**

$= \left[\frac{3\,(6+1)}{2}\right]^2 = (21)^2 = 441$

## Square Number

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $1^2$ | = 1 | $2^2$ | = 4 | $3^2$ | = 9 | $4^2$ | = 16 | $5^2$ | = 25 |
| $6^2$ | = 36 | $7^2$ | = 49 | $8^2$ | = 64 | $9^2$ | = 81 | $10^2$ | = 100 |
| $11^2$ | = 121 | $12^2$ | = 144 | $13^2$ | = 169 | $14^2$ | = 196 | $15^2$ | = 225 |
| $16^2$ | = 256 | $17^2$ | = 289 | $18^2$ | = 324 | $19^2$ | = 361 | $20^2$ | = 400 |
| $21^2$ | = 441 | $22^2$ | = 484 | $23^2$ | = 529 | $24^2$ | = 576 | $25^2$ | = 625 |
| $26^2$ | = 676 | $27^2$ | = 729 | $28^2$ | = 784 | $29^2$ | = 841 | $30^2$ | = 900 |
| $31^2$ | = 961 | $32^2$ | = 1024 | $33^2$ | = 1089 | $34^2$ | = 1156 | $35^2$ | = 1225 |
| $36^2$ | = 1296 | $37^2$ | = 1369 | $38^2$ | = 1444 | $39^2$ | = 1521 | $40^2$ | = 1600 |
| $41^2$ | = 1681 | $42^2$ | = 1764 | $43^2$ | = 1849 | $44^2$ | = 1936 | $45^2$ | = 2025 |
| $46^2$ | = 2116 | $47^2$ | = 2209 | $48^2$ | = 2304 | $49^2$ | = 2401 | $50^2$ | = 2500 |

## Square Root of 1st Ten Number

| | | |
|---|---|---|
| $\sqrt{1} = 1$ | $\sqrt{2} = 1.41421...$ | $\sqrt{3} = 1.73205$ |
| $\sqrt{4} = 2$ | $\sqrt{5} = 2.23606...$ | $\sqrt{6} = 2.44948$ |
| $\sqrt{7} = 2.64575$ | $\sqrt{8} = 2.82842$ | $\sqrt{9} = 3$ |
| $\sqrt{10} = 3.16227...$ | | |

## Square Root

| | | | |
|---|---|---|---|
| $\sqrt{1} = 1$ | $\sqrt{4} = 2$ | $\sqrt{9} = 3$ | $\sqrt{16} = 4$ |
| $\sqrt{25} = 5$ | $\sqrt{36} = 6$ | $\sqrt{49} = 7$ | $\sqrt{64} = 8$ |
| $\sqrt{81} = 9$ | $\sqrt{100} = 10$ | $\sqrt{121} = 11$ | $\sqrt{144} = 12$ |
| $\sqrt{169} = 13$ | $\sqrt{196} = 14$ | $\sqrt{225} = 15$ | $\sqrt{256} = 16$ |
| $\sqrt{289} = 17$ | $\sqrt{324} = 18$ | $\sqrt{361} = 19$ | $\sqrt{400} = 20$ |
| $\sqrt{441} = 21$ | $\sqrt{484} = 22$ | $\sqrt{529} = 23$ | $\sqrt{576} = 24$ |
| $\sqrt{625} = 25$ | $\sqrt{676} = 26$ | $\sqrt{729} = 27$ | $\sqrt{784} = 28$ |
| $\sqrt{841} = 29$ | $\sqrt{900} = 30$ | $\sqrt{961} = 31$ | $\sqrt{1024} = 32$ |
| $\sqrt{1089} = 33$ | $\sqrt{1156} = 34$ | $\sqrt{1156} = 35$ | $\sqrt{1296} = 36$ |
| $\sqrt{1369} = 37$ | $\sqrt{1444} = 38$ | $\sqrt{1521} = 39$ | $\sqrt{1600} = 40$ |
| $\sqrt{1681} = 41$ | $\sqrt{1764} = 42$ | $\sqrt{1849} = 43$ | $\sqrt{1936} = 44$ |
| $\sqrt{2025} = 45$ | $\sqrt{2116} = 46$ | $\sqrt{2209} = 47$ | $\sqrt{2304} = 48$ |
| $\sqrt{2401} = 49$ | $\sqrt{2500} = 50$ | | |

**• Square number:**

In mathematics, a square number or perfect square is an integer that is the square of an integer; in other words, it is the product of some integer with itself. For example, 9 is a square number, since it can be written as 3 x 3.

**Some Important Points**

1. A number n is a prefer. square if n = m for some integer m
2. A perfect square number is never negative
3. A square number never ends in 2, 3, 7or 8
4. The number of zeros at the end of the perfect square is even.
5. The square of an even number is odd and odd number is even.
6. A perfect square number never leaves remainder 0 or "I on division by 3.
7. If a number has a square root then its unit digit must be 0, 1, 4, 5, 6, or 9.

| Unit digit of square | 0 | 1 | 4 | 5 | 6 | 9 |
|---|---|---|---|---|---|---|
| Unit digit of square root | 0 | 1 or 9 | 2 or 8 | 5 | 4 or 6 | 3 or 7 |

8. For every natural number n, $(n+1)^2 - n^2 = (n+1) + n$

   For example: $9^2 - 8^2 = 9 + 8$

   81 – 64 = 17
9. The square of natural number n is equal to the sum of first an odd natural numbers

   Eg: $1 = 1^2$ $1 + 3 = 1^2$ $1 + 3 + 5 = 3^2$
10. For any natural number m > 1, $(2m, m^2 - 1, m^2 + 1)$ is a Pythagoras triplet

## AVERAGE

An average or more accurately an arithmetic mean is, in crude terms, the sum of n different data divided by n.

For example, if a batsman scores 35, 45 and 37 runs in first, second and third innings respectively, then his average runs in 3 innings is equal to $\frac{35+45+37}{3} = \mathbf{39}$ runs.

Therefore, the two mostly used formulas in this chapter are:

Average $= \frac{\text{Total of data}}{\text{Total of data}}$ $or = \frac{\textit{Sum of observation}}{\textit{No.of observation}}$

And, Total = Average × No. of Data

**Direct Formula**

Age of entrant = New average + No. of old members × Increase

**Direct Formula:**

Weights of new person = weight of removed person + No. of person × increases in average

**Direct Formula:**

Number of passed candidates

$= \frac{\text{Total candidate (Total average – Failed average)}}{\text{Passed average – Failed average}}$

And number of failed candidates

$= \frac{\text{Total candidates (Passed average – Total average)}}{\text{Passed average – Failed average}}$

**Average Related to speed**

**Therorem : If a person travels a distance at a speed of x km/hr and the same distance at a speed of y km/hr, then the average speed during the whole journey is given by $\frac{2xy}{x+y}$**

If half of the journey is travelled at a speed of x km/hr and the next half at a speed of y km/hr, then average speed during the whole journey is $\frac{2xy}{x+y}$ **km/hr**.

If a man goes to a certain place at a speed of x km/hr and returns to the original place at a speed of y km/hr. then the average speed during up-and down journey is $\frac{2xy}{x+y}$ **km/hr**.

**Theorem: If a person travels three equal distances at a speed of x km/hr, y km/hr and z km/hr respectively, then the average speed the whole journey is $\frac{3xyz}{xy+yz+xz}$ km/hr.**

Proof : Let the three equal distance be A km.

Time taken at the speed of x km/hr = $\frac{A}{x}$ **hrs**.

Time taken at the speed of y km/hr = = $\frac{A}{y}$ **hrs**.

Time taken at the speed of z km/hr = = $\frac{A}{z}$ **hrs**.

Total distance travelled in time = $\frac{A}{x}+\frac{A}{y}+\frac{A}{z} = \mathbf{3A\ km}$

Average speed during the whole journey = $\frac{3A}{\frac{A}{x}+\frac{A}{y}+\frac{A}{z}} = \frac{3xyzA}{xyz+Axz+Axy} = \frac{3xyz}{xy+yz+xz}$ km/hr

**Unlike terms :** - Terms that do not contain the same power of the same varies able are called unlike terms.

**Eg : - 3x and 3y, 3x and 6x²**

**Factor theorem** : P(x) is a polynomial and is a real number, if, p (a) = o then x – a is a factor of P (x)

## BASIC ALGEBRAIC EXPRESSIONS

We know that the algebraic identities of second degree and these idens titles can be used to factorise quadratic polynomials.

A polynomial is said to be cubic polynomial if its degree is three

The algebraic identities used in factorizing a third degree polynomials are :


i. $(a+b)^2 = a^2+2ab+b^2$

ii. $(a+b)^2 = (a-b)^2+4ab$

iii. $(a-b)^2 = a^2-2ab+b^2$

iv. $(a-b)^2 = (a+b)^2-4ab$

v. $a^2-b^2 = (a+b)(a-b)$

vi. $a^2+b^2 = (a+b)^2-2ab$

vii. $a^2+b^2 = (a-b)^2+2ab$

viii. $(x+a)(x+b) = x^2+(a+b)x+ab$

ix. $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

x. $(a+b)^3 = a^3+b^3+3ab(a+b)$

xi. $(a-b)^3 = a^3-b^3-3ab(a-b)$

xii. $a^3+b^3 = (a+b)(a^2-ab+b^2)$

xiii. $a^3-b^3 = (a-b)(a^2+ab+b^2)$

xiv. $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

xv. $(x+a)(x+b) = x^2+(a+b)x+ab$

xvi. $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

xvii. $a+b = \frac{a^3+b^3}{a^2-ab+b^2}$

xviii. $a-b=\dfrac{a^3-b^3}{a^2+ab+b^2}$

xix. $a+b+c=\dfrac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca}$

xx. If a + b +c = 0 then, $a^3+b^3+c^3=3abc$

xxi. $ab=\dfrac{(a+b)^2}{2}-\dfrac{(a^2+b^2)}{2}$

xxii. $a^m \times a^n = a^{m+n}$

xxiii. $a^m \div a^n = a^{m-n}$

xxiv $(a^m)^n = a^{mn}$

xxv. $(a \times b)^n = a^n \times b^n$

xxvi. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

xxvii. $\dfrac{a^n}{a^n} = a^{n-n} = a^0 = 1$

xxviii. $a^n = a \times a \times a ... n$

xxix. $a^{-n} = \dfrac{1}{a^n}$

xxx. $a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m}$

xxxi. $a^{1/n} = \sqrt[n]{a}$.

xxxii. $a^m \times b^m = (a \times b)^m$

xxxiii. n $(A \cup B)$ = n(A) + n(B) - N $(A \cap B)$

xxxiv. n $(A \cup B \cup C)$ = n(A) + n(B)+ n(C)- n$(A \cap B)$ - n $(B \cap C)$ - n$(A \cap C)$ + $(A \cap B \cap C)$

## PERCENTAGE

### Percent

The term percent means 'for every hundred'. If can be defined as: "A fraction whose denominator is 100 is called percentage, and the numerator or the fraction is called the rate per cent."

### Percentage value

| | | | |
|---|---|---|---|
| $\frac{1}{2}=50\%$ | $\frac{1}{3}=33\frac{1}{3}\%$ | $\frac{1}{4}=25\%$ | $\frac{1}{5}=20\%$ |
| $\frac{1}{6}=16\frac{2}{3}\%$ | $\frac{1}{7}=14\frac{2}{7}\%$ | $\frac{1}{8}=12\frac{1}{2}\%$ | $\frac{1}{9}=11\frac{1}{9}\%$ |
| $\frac{1}{10}=10\%$ | $\frac{1}{11}=9\frac{1}{11}\%$ | $\frac{1}{12}=8\frac{1}{3}\%$ | |

| | | | |
|---|---|---|---|
| $4\%=\frac{1}{25}$ | $5\%=\frac{1}{20}$ | $10\%=\frac{1}{10}$ | $16\%=\frac{4}{25}$ |
| $20\%=\frac{1}{5}$ | $25\%=\frac{1}{4}$ | $40\%=\frac{2}{5}$ | $50\%=\frac{1}{2}$ |
| $60\%=\frac{3}{5}$ | $75\%=\frac{3}{4}$ | $80\%=\frac{4}{5}$ | $100\%=1$ |
| $120\%=\frac{6}{5}$ | $125\%=\frac{5}{4}$ | $6\frac{1}{4}\%=\frac{1}{16}$ | $12\frac{1}{2}\%=\frac{1}{8}$ |
| $37\frac{1}{2}\%=\frac{3}{8}$ | $87\frac{1}{2}\%=\frac{7}{8}$ | $8\frac{1}{3}\%=\frac{1}{12}$ | $16\frac{2}{3}\%=\frac{1}{6}$ |
| $33\frac{1}{3}\%$ | $66\frac{2}{3}\%=\frac{5}{8}$ | $150\%=\frac{2}{3}$ | |

### Some more points on Percentage

1. If two values are respex% & y% more than a third value, then the first is the
   - $\dfrac{100+x}{100+y} \times 100\%$ of the second .
2. If A is x % of C & B in y% of C, then
   - A is $\dfrac{x}{y} \times 100\%$ of B
3. x% of a quantity is taken by the first, y% of the remainning is taken by the second & Z% of the remaining is taken by third person, now, if A is left in the fund, then there was
   - $\dfrac{A \times 100 \times 100 \times 100}{(100-x)(100-y)(100-z)}$ in the begining
4. x% of a quanting is added. Again y% of the increased quantity is added. Again, Z% of the increased quantity ia added. Now it becomes A, then the initial amount is given by
   - $\dfrac{A \times 100 \times 100 \times 100}{(100+x)(100+y)(100+z)}$
5. The Population of a town is P. It increases by x% during first year, increases y% during the second year and againt increases by Z% during the third year. The population after third year will be
   - $\dfrac{P \times (100+x)(100+y)(100+z)}{100 \times 100 \times 100}$
6. When the population decreases by y% during the second year, while for the first and third years, it follows the same, the population after 3 years will be
   - $\dfrac{p(100+x)(100-y)(100+z)}{100 \times 100 \times 100}$
7. If the Price of a commodity increases by r%, then the reduction in consumption so as not to increase the expenditure, is

- $\left(\frac{r}{100+r}\times 100\right)\%$

8. If the Price of a commodity decreases by r%, then increases in consumption, so as not to decrease expenditure on this item, is

- $\left(\frac{r}{100-r}\times 100\right)\%$

9. If the value of a number is first increased by x% and later decreased by x% , the net change is always a decrease which is equal to

- x% of x or $\frac{x^2}{100}$

10. If the value is first increased by x% and the decreased by y% then there is increase or decrease

- $\left(x-y-\frac{xy}{100}\right)\%$

11. If the value is increased sucessively by x% and y% then the final increase is given by

- $\left[x+y+\frac{xy}{100}\right]\%$

12. The passing marks in an examination is x% if a candidate who scores y marks fail by z marks, then the maximum marks,

- $M=\frac{100(y+z)}{x}$

3. A candidate scoring x% in an examination fails by 'a' marks while another candidate who scores y% marks get 'b' marks more than the minimum required pass marks. Then, the maximum marks for that examination are

- $M=\frac{100(a+b)}{y-x}$

4. In measuring the sides of a rectangle, one side is taken x% in excess and the other y% in deficit. The error percent in area calculated from the Measurement is

- $x-y-\frac{xy}{100}$ in excess or deficit according to the **+ve** or **-ve** sign.

15. If the sides of a triangle, rectangle, square, circele, rhombus (or any 2-dimentional figure) are increased by x%, its area is increased by

- $\frac{x(x+200)}{100}\%$ or $\left[2x+\frac{x^2}{100}\%\right]$

16. If all the sides of a triangle are increased by x% then its area increases by

- $\frac{x(x+200)}{100}\%$ or $2x+\frac{x^2}{100}$

*[Note: It is same as for the triangle.]*

17. In an examination , x% failed in English and y% failed in maths. If Z% of students failed in both the subjects, the percentage of students who passed in both the subjects is
   - 100- (x + y + z) or (100 - x) + (100 - y) + z

## PROFIT AND LOSS

**C.P : - Cost Price**
**S.P : - Selling Price**

**Cost Price:-** The amount paid to purchase an article or the price at which an article is made is known as its cost price.

**Selling Price:-** The Price at which an article is sold is known as its selling price.

**Profit: -** If the selling Price (S.P) of an article is greater than the cost price (C.P), the difference between the selling price and cost price is called profit.

**If S.P > C.P, then**
**Profit = S.P – C.P**

**Loss : -** If the selling price (S.P) of an article between the cost price (C.P) and the selling price (S.P) is called loss.

If C.P > S.P, then
Loss = C.P – S.P

Sell / sold/ Selling means S.P.
Buy/Cost/Purchase means C.P.

**S.P = C.P + Profit**
**S.P = C.P – Loss**
**C.P = S.P – Profit**
**C.P= S.P + Loss**

S.P = $\frac{CP\times(100+G\%)}{100}$

S.P = $\frac{CP\times(100-L\%)}{100}$ $\left(\begin{matrix}G\%=Gain\%\\ L\%=Loss\%\end{matrix}\right)$

C.P = $\frac{SP\times 100}{100+G\%}$

C.P = $\frac{SP\times 100}{100-L\%}$

## DISCOUNT

### Marked Price/List Price (M.P)

While buying goods that on every article there is a Price marked. This price is known as the marked price of the article.

### Discount

Sometimes shopkeepers offer a certain percent of rebate on the marked Price for cash payments. This rebate is known as discount.

Discount = MP – SP

Rate of Discount (Discount%) = $\frac{Discount}{MP}\times 100$

S.P = M.P – Discount

SP = MP– $\frac{MP\times Discount\%}{100}$

SP=MP× $\left[1-\frac{Discount\%}{100}\right]$

SP = MP × $\left[\frac{100-Discount\%}{100}\right]$

MP = $\frac{100\times SP}{100-Discount\%}$

More formulas based on different possibility

- % gain = $\frac{\text{Error}}{\text{Truevalue- Error}}\times 100$
- % gain = $\frac{\text{True Weight- Falseweight}}{\text{Falseweight}}\times 100$
- Total percentage profit = $\frac{\%profit+\%less\ \ in\ weight}{100\text{-}\%\ Less\ \ in\ weight}\times 100$
- Cost = $\frac{more\ \ gain\ \times 100}{Difference\ \ in\ percentage\ \ profit}$
- When cost Price & seling price are reduced by the same amount (Say A) then
- Cost Price = $\frac{[\text{initial profit }\%+\text{increase in profit }\%\ ]\times A}{\text{increase in Profit}}$
- Selling Price = $\frac{More\ \ rupees\ (100+\%final\ \ gain)}{\%\ gain\ +\ \%\ loss}$
- If x Part is sold at m% Profit, y part is sold at n% Profit, z part is sold at p% Profit and P is earned as overall profit then the value of total consignment.
- $=\frac{p\times 100}{xm+ny+pz}$
- Percentage Profit =

$$\frac{\text{first part}\times\%\text{ profit on first part}+\text{second part}\times\%\text{ profit on second part}}{\text{Total of two part}}$$

- A man Purchases a certain no, of articles at x a rupee and the same no at y a rupee. He mixes them together and sells them at z a rupee. Then his gain or loss%

$=\left[\frac{2xy}{z(x+y)}-1\right]\times 100$ according to +ve or -ve sign.

If a trademan marks his goods at x% above his cost price and allows purchases a discount of y% for cash, then there is $\left(x-y-\frac{xy}{100}\right)$ % profit or loss according to +ve or -ve sign.

When each of the two commodities is sold at the same price. and profit of p% in made on the first & a loss of L% is made on the second, then the percentage gain or loss.

$=\frac{100(P-L)-2PL}{(100+P)(100-L)}$ according to +ve or -ve sign.

## TIME AND WORK

if

| | | |
|---|---|---|
| '$M_1$' Persons can do | : | "$M_2$" |
| '$W_1$' Work in | : | "$W_2$" |
| '$D_1$' days and | : | "$D_2$" |

Then the general formula in the relationship of $M_1 D_1 W_2 = M_2 D_2 W_1$

1. More men less days and conversely more days less men.
2. More men more work and conversely more work men.
3. More days more work and conversely more work days.

Also include the working hours $(T_1\ \&\ T_2)$ for the two groups, then the relationship is.

$$M_1\ D_1\ T_1\ W_2 = M_2\ D_2\ T_2\ W_1$$

Again include, the efficiency of the persons $E_1\ \&\ E_2$ in two groups is different the the realtionship is

$$M_1\ D_1\ T_1\ E_1\ W_2 = M_2\ D_2\ T_2\ E_1\ W_1$$

**Eg(1):- 5 men can prepare 10 toys in 6 days working 6 hrs a day Then, in how many days can 12 men prepare 16 toys working 8 hrs a day?**

Sol. $M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_1$
$5\times 6\times 6\times 16 = 12\times D2\times 8\times 10$

$D_2 = \frac{5\times 6\times 6\times 16}{12\times 8\times 10} = 3$ days

2. If A can do a piece of work in x days and B can do it in y days then A and B working together will do the same work in $\frac{xy}{x+y}$ days.
3. If A and B together can do a piece of work in x days and A alone can do it in y days, then B alone can do the work in $\frac{xy}{x-y}$ days.

If A, B, & C can do a work in x, y & z days respectively, then all of them working together can finish the work in $\frac{xyz}{xy+yz+xz}$ days

## WORK & WAGES

Wages are distributed in proportion to the work done and in indirect (or inverse) proportion to the time taken by the individual.
There are two methods: -
Eg. A can do a work in 6days and B can do the same work in 5 days. The contract for the work is Rs. 220. How much shall B get If both of them work together.
Methods I : -
A's 1 day's work = $\frac{1}{6}$; B's 1 days work = $\frac{1}{5}$
Ratio of their wages = $\frac{1}{6}:\frac{1}{5} = 5:6$
B's share = $\frac{220}{5+6} \times 6$ = Rs. 120
Methods II : -
As wages are distributed in inverse proportion of number of days, their share should be in the ratio 5 : 6
B's share = $\frac{220}{11} \times 6$ = Rs. 120

## PIPES AND CISTERNS

**Pipes & Cisterns:** These problems are almost the same as those of Time and work problems. Thus, if a pipe fills a tank in 6 hrs, then the pipe fills 1/6th of the tank in hour.
There is one difference that pipes & cisterns problems is that there are outlets as well as inlets. Thus, there are agents (the outlets) which perform negative work too. The rest of the process is almost similar.

**Inlet**

A pipe connected with a tank (or a cistern or a reservoir) is called an inlet, if it fills it.

**Outlet**

A pipe connected with a tank is called an outlet, if it empties it.

**Formulae**

I. If a pipe can fill a tank in x hours, then the part filled in 1 hour = $\frac{1}{x}$
II. If a pipe can empty a tank in y hours, then the part of the full tank 1 empited in 1 hour = $\frac{1}{y}$
III. If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours, then the net part filled in 1 hour, when both the pipes are opened = $\left(\frac{1}{x}-\frac{1}{y}\right)$
time taken to fill the rank, when both the pipes are opened = $\frac{xy}{y-x}$
IV. If a pipe can fill a tank in x hours and another can fill the same tank in y hrs, then the net part filled in 1 hr, when both the pipes are opened = $\left(\frac{1}{x}+\frac{1}{y}\right)$
time taken to fill the tank = $\frac{xy}{y+x}$
V. If a pipe fills a tank in x hrs & another fills the same tank in y hrs, but a third one empties the full tank in z hrs, and all of them are opened together, the net part filled in 1 hr = $\left[\frac{1}{x}+\frac{1}{y}-\frac{1}{z}\right]$
time taken to fill the tank = $\frac{xy}{y+x-zy}$ $hrs$.
VI. A pipes can fill a tank in x hrs. Due to a leak in the bottom it is filled in y hrs. If the tank is full, the time taken by the leak to empty the tank = $\frac{xy}{y-x}$ $hrs$.

Eg. Two pipes A & B can fill a tank in 45 hrs & 36 hrs. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?
Sol$^n$. Part filled by A alone in 1 hr = $\frac{1}{45}$
Part filled by B alone in 1 hr = $\frac{1}{36}$
Part filled by (A +B) in 1 hr = $\left(\frac{1}{45}+\frac{1}{36}\right) = \frac{9}{180} = \frac{1}{20}$
Hence, both the pipes together will fill the tank in 20 hours.

## TIME AND DISTANCE

i. Speed = $\frac{\text{Distance}}{\text{Time}}$
ii. Time = $\frac{\text{Distance}}{\text{Speed}}$
iii. Distance = Speed x Time

(where D=distance, S=Speed, T=Time)

iv. x Km/hr = $\left(x \times \frac{5}{18}\right)$ km/hr
v. x metres/ sec = $\left(x \times \frac{18}{5}\right)$ km/hr

vi. If speed of a body is changed in the ratio a : b, then the ratio of the time changes in the ratio b : a
vii. If a certain distance is covered at x km/hr and the same distance is covered at y km/hr then the average speed during the whole journey is $\frac{2xy}{x+y}$ km/hr
viii. It two persons A & B start at the same time in opposite directions from two points and after passing each other they complete the journey in 'a' & 'b' hrs, then
A's speed: B's Speed = $\sqrt{b} : \sqrt{a}$

**Some Direct Formulas :**

1. Required distance : $\frac{\textit{Product of two speed}}{\textit{Difference of two speed}} \times$ Difference b/w arriva times
2. Required distance : Total time taken × $\frac{\textit{Product of the two speed}}{\textit{Addition of the two speed}}$

3. Distance = $\frac{2 \times \text{Time} \times S_1 \times S_2}{S_1 + S_2}$

Where, S1 = Speed during first half and
S2 = Speed during second half of journey

4. Meeting points distance from starting point

$$= \frac{S_1 \times S_2 \times \text{Difference in the time}}{\text{Difference in Speed}}$$

where $S_1$ and $S_2$ are the speeds of the first & the second trains respectively

5. When the ratio of speeds of A to B is a : b, then

Distance travelled by A = 2 x Distance of two points $\left(\frac{a}{a+b}\right)$ &

Distance travelled by B = 2 x Distance of two points $\left(\frac{b}{a+b}\right)$

6. Distance

$$= \frac{\text{Multiplication of Speeds}}{\text{Difference of Speeds}} \times \text{Distance in time to cover the distance}$$

7. Time of rest per hour = $\frac{\text{Difference of Speed}}{\text{Speed without Sto page}}$

8. Distance = Total Time × $\frac{\text{Multiplication of two Speed}}{\text{Sum of Speed}}$

9. If $\frac{1}{3}$rd of the distance is covered at x km/hr next $\frac{1}{3}$rd is covered at y km/hr and next $\frac{1}{3}$rd is covered at z km/hr then the average speed is km/hr. $\frac{3xyz}{xy + yz + zx}$

10. If a train crosses a pole means it crosses it's own length.

11. If a train crosses a bridge means it crosses (it's own length + length of the bridge)

12. When a train is passing another train completely (whether moving in the same direction or in opposite directions), it has to cover a distance equal to the sum of the lengths of the two trains.

13. If two moving trains (in the same direction) crosses each other then relative speed = (speed of the faster trains—speed of the slower train)

14. If two trains are moving in the opposite directions then the relative speed = (speed of the Ist train + Speed of the 2nd train)

15. If without stoppage a train covers a distance at an average speed of $x_1$km/hr and with stoppage it covers a distance at an average speed of $x_2$ km/hr then it's stoppage time per hour is $t_2 \left(\frac{x^2 - x^2}{x_1}\right)$ where $t_2$ = stoppage time per hour.

## Some Important points based on Trains

1. When two trains are moving in opposite directions their speeds should be added to find the relative speed.
2. When they are moving in the same direction the relative speed is the difference of their speeds.
3. When a train passes a platform it should travel the length equal to the sum of the lengths of train & platform both.

## Direct formulas based on Trains

Distance = Difference in distance x $\frac{\text{sum of speed}}{\text{Difference in Speed}}$

Length of the train

$\frac{\text{Length of Platform}}{\text{Difference in time}} \times$ Time taken cross a stationary pole or man

Length of the train= $\frac{\text{Difference in speed of two men} \times T_1 \times T_2}{(T_1 - T_2)}$

*(Note:- The slower person takes more time than the faster person to cross the train, so it was just opposite)*

Length of the train= $\frac{\text{Difference in speed} \times \text{Multiplica tion of times}}{\text{Difference in time}}$

Length of the train = $\frac{\text{Time to pass a pole} \times \text{Length of the Platform}}{\text{Difference in time to cross a pole \& Platform}}$

- Speed of the other train

= Speed of the first train × $\sqrt{\frac{\text{Time taken by } 1^{st} \text{ train after meeting}}{\text{Time taken by } 2^{nd} \text{ train after meeting}}}$

## STREAMS

**Upstream: -** If the boat moves against the stream then it is called upstream.

**Downstream: -** If it moves with the stream, it is called downstream.

**Note: -** If the speed of the boat (or the swimmer) is x and If the speed of the stream is y then,

1. while upstream the effective speed of the boat = x - y
2. while downstream the effective speed of the boat = x+ y

**Theorems based on streams (upstream & down streams)**

i. If x km per hour be the man's rate in still water, & y km per hour the rate of the current. Then
   - x + y = man's rate with current.
   - x - y = man's rate against current.

   Adding & Subtracting and then dividing by 2
   - x = ½ (man's rate with current + his rate against current)
   - y = ½ (man's rate with current - his rate against current)

**Facts:-**

1. A man's rate in still water is half the sum of his rates with and against the current
2. The rate of the current is half the difference between the rates of the man with and against the current.

**Theorems: -**

1. A man can row x km/hr is still water. If in a stream which is flowing at y km/hr, it takes him z hrs to row to a place and back, the distance between the two places is $\frac{z(x^2+y^2)}{2x}$
2. A man rows a certain distance downstream in x hours and returns the same distance in y hrs. If the stream flows at the rate of z km/hr then the speed of the man in still water is given by = $\frac{z(x+y)}{y-x}$ km/hr

## SIMPLE AND COMPOUND INTEREST

$$CI = A - P$$

When interest is compounded annually:

$$\text{Amount} = P\left[1+\frac{r}{100}\right]^t$$

$$\text{or, } A = P\left[1+\frac{R}{100}\right]^n$$

When interest is compounded Half-yearly:

$$\text{Amount} = P\left[1+\frac{\frac{r}{2}}{100}\right]^{2t} = P\left[1+\frac{r}{200}\right]^{2t}$$

When interest is compounded quaterly:

$$\text{Amount}: P\left[1+\frac{\frac{r}{4}}{100}\right]^{4t} = P\left[1+\frac{r}{400}\right]^{4t}$$

When rate of interest is is $r_1$%, $r_2$% & $r_3$% for 1 st year, 2nd year & 3rd year,

$$\text{Amount} = P\left[1+\frac{r_1}{100}\right]\times\left[1+\frac{r_2}{100}\right]\times\left[1+\frac{r_3}{100}\right]$$

If certain sum becomes 'm' times in 't' years, the rate of compound interest

$$r = 100\left[(m)^{\frac{1}{t}} - 1\right]$$

- Given CI, to find SI

$$S.I. = \frac{rt}{100\left[\left(1+\frac{r}{100}\right)^t - 1\right]} \times CI$$

- When difference between the compound interest & simple interest on a certain sum of money for 2 years at r% rate is □x, then the sum is given by:

$$\text{Sum} = \frac{\text{Difference}\times 100\times 100}{\text{Rate}\times\text{Rate}} = \frac{x(100)^2}{r^2} = x\left(\frac{100}{r}\right)^2$$

- And when sum is given & difference b/w SI & CI is asked, then:-

$$\text{Difference} = \text{sum}\left(\frac{r}{100}\right)^2$$

- If the difference b/w CI & SI on a certain sum for 3 years at r% is Rs. x, the Sum will be

$$\frac{\text{Difference}\times(100)^3}{r^2(300+r)}$$

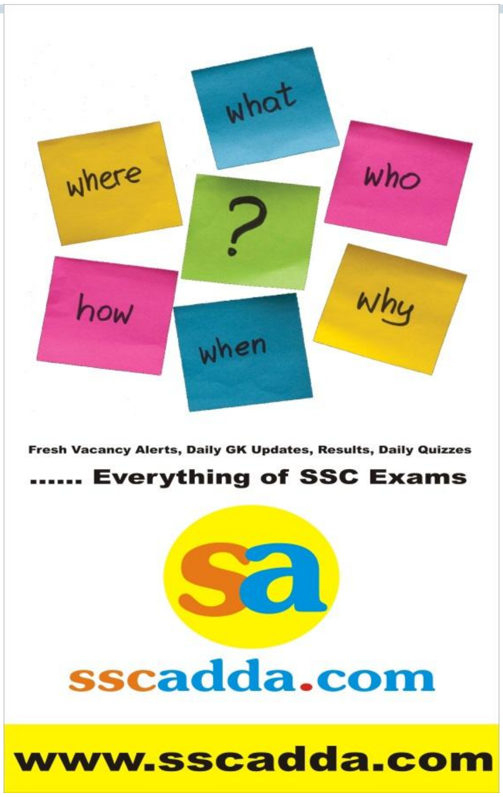
and if the sum is given & the difference is asked, then

$$\text{Difference} = \frac{Sr^2(300+r)}{(100)^3}$$

Where, Sum = S

- When interest is compounded Annually

$$\text{Amount} = P\left[1+\frac{r}{100}\right]^t$$



When interest is compounded half yearly

$$\text{Amount} = P\left[1+\frac{r/2}{100}\right]^{2t}$$

- When interest is compounded Quarterly

$$\text{Amount} = P\left[1+\frac{r/4}{100}\right]^{4t}$$

- When interest is compounded biannually.

$$\text{Amount} = P\left[1+\frac{2r}{100}\right]^{\frac{t}{2}}$$

- Equal annual instalment to pay the borrowed amount at compound interest if
  A = value of each equal instalment  R = rate of interest
  n = number of instalments per years  t= number of years
  B= Borrowed amount.

**then,**

$$B = A\left[\frac{100}{100+R}+\left(\frac{100}{100+r}\right)^2+\ldots+\left(\frac{100}{100+R}\right)^{n\times t}\right]$$

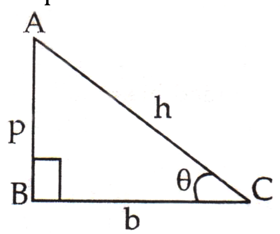
**or,**

$$B = A\left[1-\left(\frac{100}{100+R}\right)^{nt}\right]\frac{R}{100}$$

$$\text{or, } A = \left[\frac{B}{1-\left(\frac{100}{100+R}\right)^{nt}}\right]\frac{R}{100}$$

## TRIGONOMETRY

**Pythagoras Theorem:** - In a right angled the square of the hypotenuse is sum of the squares of the base of the perpendicular. h2 = p2 + b2




### A. Trigonometric Ratios

The ratios of the sides of a right – angled triangle with respect to its angles are called trigonometric ratios.
AB = Perpendicular (P)
BC = Base (B) and AC = Hypotenuse (H)
Hint
Some People Have, Curly Black Hair, Turn Permentely Brown

1. $\sin\theta = \frac{P}{H}$

   $\sin\theta = \frac{1}{\text{cosec}\theta} \Rightarrow \text{cosec}\theta = \frac{H}{P}$

   $\therefore \sin\theta \times \text{cosec}\theta = 1$

2. $\text{Cos}\theta = \frac{B}{H}$

   $\text{Cos}\theta = \frac{1}{\sec\theta} = \sec\theta = \frac{H}{B}$

   $\therefore \cos\theta \times \sec\theta = 1$

3. $\tan\theta = \frac{P}{B}$

   $\tan\theta = \frac{1}{\cot\theta} \Rightarrow \cot\theta = \frac{B}{P}$

   $\therefore \tan\theta \times \cot\theta = 1$

4. $\tan\theta = \frac{\sin\theta}{\cos\theta},\ \cot\theta = \frac{\text{Cos}\theta}{\text{Sin}\theta}$

5. $\sin^2\theta + \cos^2\theta = 1$

   $\therefore \sin^2\theta = 1 - \cos^2\theta,$

   $\cos^2\theta = 1 - \sin^2\theta$

6. $\sec^2\theta - \tan^2\theta = 1 \therefore \sec^2\theta = 1 + \tan^2\theta, \tan^2\theta = \sec^2\theta - 1$

7. $\text{cosec}^2\theta - \cot^2\theta = 1 \Rightarrow \therefore \text{cosec}^2\theta = 1 + \cot^2\theta \Rightarrow \cot^2\theta = \text{cosec}^2\theta - 1$

8. $\sin\theta = \sqrt{1-\cos^2\theta}$  9. $\cos\theta = \sqrt{1+\tan^2\theta}$

10. $\text{cosec}\theta = \sqrt{1+\cot^2\theta}$  11. $\cos\theta = \sqrt{1-\sin^2\theta}$

12. $\tan\theta = \sqrt{\sec^2\theta - 1}$  13. $\cot\theta = \sqrt{\text{cosec}^2\theta - 1}$

1. $\sin\theta = \cos(90-\theta)$  2. $\cos\theta = \sin(90-\theta)$
3. $\tan\theta = \cot(90-\theta)$  4. $\cot\theta = \tan(90-\theta)$
5. $\sec\theta = \text{cosec}(90-\theta)$  6. $\text{cosec}\theta = \sec(90-\theta)$
7. $\tan\theta \times \tan(90-\theta) = 1$  8. $\cot\theta \times \cot(90-\theta) = 1$

| | 0° | 30° | 45° | 60° | 90° |
|---|---|---|---|---|---|
| Cosec Sin | $\sqrt{\frac{0}{4}}=0$ | $\sqrt{\frac{1}{4}}=\frac{1}{2}$ | $\sqrt{\frac{2}{4}}=\frac{1}{\sqrt{2}}$ | $\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$ | $\sqrt{\frac{4}{4}}=1$ |
| Sec Cos | $\sqrt{\frac{4}{4}}=1$ | $\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$ | $\sqrt{\frac{2}{4}}=\frac{1}{\sqrt{2}}$ | $\sqrt{\frac{1}{4}}=\frac{1}{2}$ | $\sqrt{\frac{0}{4}}=0$ |
| Cot Tan | $\sqrt{\frac{0}{4}}=0$ | $\sqrt{\frac{1}{3}}=\frac{1}{\sqrt{3}}$ | $\sqrt{\frac{2}{2}}=1$ | $\sqrt{\frac{3}{1}}=\sqrt{3}$ | $\sqrt{\frac{4}{0}}=\text{n.d}$ |

Note: Circle are the Reciprocal of the given value

Eg: $\sin 30 = \frac{1}{2}$ $\quad$ $\tan 60 = \sqrt{3}$

$\text{Cosec}\,30 = \frac{2}{1}$ $\quad$ $\cot 60 = \frac{1}{\sqrt{3}}$

## D. Important Formula

1. $\sin(A+B) = \sin A.\cos B + \cos A.\sin B$
2. $\sin(A-B) = \sin A.\cos B - \cos A.\sin B$
3. $\cos(A+B) = \cos A.\cos B. - \sin A.\sin B$
4. $\cos(A-B) = \cos A.\cos B. + \sin A.\sin B$
5. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A.\tan B}$
6. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A.\tan B}$
7. $\cot(A+B) = \dfrac{\cot A.\cot B - 1}{\cot B + \cot A}$
8. $\cot(A-B) = \dfrac{\cot A.\cot B + 1}{\cot B - \cot A}$
9. $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A.\tan B.\tan C}{1 - \tan A.\tan B - \tan B.\tan C - \tan C.\tan A}$
10. $\sin(A+B).\sin(A-B)$

    $= \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$
11. $\cos(A+B).\cos(A-B)$

    $= \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$
12. $\sin 2A = 2\sin A.\cos A. = \dfrac{2\tan A}{1+\tan^2}$
13. $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$

    $= 2\cos^2 A - 1 = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$
14. $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$
15. $2\sin^2 A = 1 - \cos 2A$
16. $2\cos^2 A = 1 + \cos 2A$
17. $\sin 3A = 3\sin A - 4\sin^3 A$
18. $\cos 3A = 4\cos^3 A - 3\cos A$
19. $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$
20. $\sin(A+B) + \sin(A-B) = 2\sin A.\cos B$
21. $\sin(A+B) - \sin(A-B) = 2\cos A.\cos B$
22. $\cos(A+B) + \cos(A-B) = 2\cos A.\cos B$
23. $\cos(A-B) - \cos(A+B) = 2\sin A.\sin B$
24. $\sin C + \sin D = 2\sin\dfrac{C+D}{2}.\cos\dfrac{C-D}{2}$
25. $\sin C - \sin D = 2\cos\dfrac{C+D}{2}.\sin\dfrac{C-D}{2}$
26. $\cos C + \cos D = 2\cos\dfrac{C+D}{2}.\cos\dfrac{C-D}{2}$
27. $\cos C - \cos D = -2\sin\dfrac{C+D}{2}.\sin\dfrac{C-D}{2}$
28. $\sin\left(\sin^{-1} x\right) = x, -1 \le x \le 1$

    $\sin^{-1}(\sin x) = x, -\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$

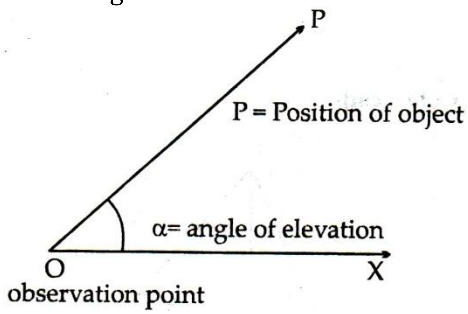
    $\cos(\cos^{-1} x) = x, -1 \le x \le 1$

    $\tan\left(\tan^{-1} x\right) = x, -\infty < x < \infty$

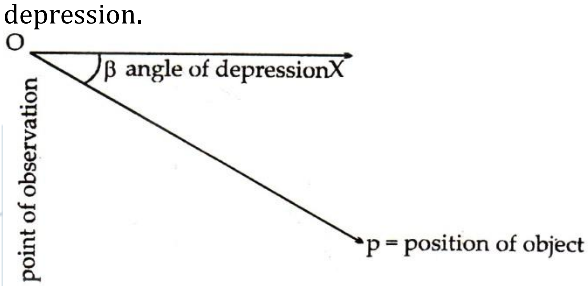
    $\tan - 1(\tan x) = x, -\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}\,etc.$

## HEIGHT & DISTANCE

**1.** If the position of the object is above the position of the observation then the angle made by the line joining object and observing point with the horizontal line drawn at the observation point is called angle of elevation.




**2.** If the position of the object is below the position of the observation the angle made by the line joining object and observation point with the horizontal line drawn at the observation point is called angle of depression.




**3.** The angle of elevation the top of a tower, standing on a horizontal plane, from a point A is, After walking a distance 'd' metres towards the foot of the tower, the angle of elevation is found to be $\beta$

The height of the tower h =

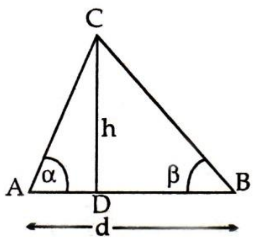
Or, h = $\frac{d}{\text{Cot}\alpha\ \cot\beta}$

Where $\overline{AB}$ = d

**4.** If the Points of observation A and B lie on either side of the tower, then height of the tower.

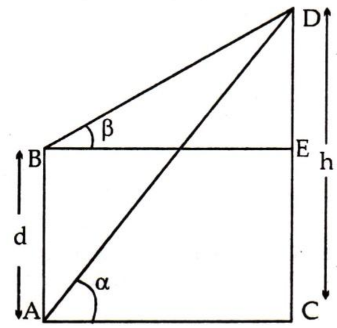
$$h = \frac{d\ \sin\alpha\ \sin\beta}{\sin(\alpha+\beta)}\ \text{where}\ \overline{AB} = d$$

$$\text{(or) } h = \frac{d}{\text{Cot}\alpha + \cot\beta}$$



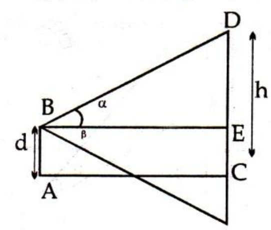

**5.** The angles of elevation of the top of a tower from the bottom and top of a building of height 'd' metres are $\beta$ and $\alpha$ respectively. The height of the tower is

$$h = \frac{d \sin\beta \cos\alpha}{\sin(\beta-\alpha)} \text{ metres (or) } h = \frac{d \cot\alpha}{\cot\alpha - \cot\beta o}$$



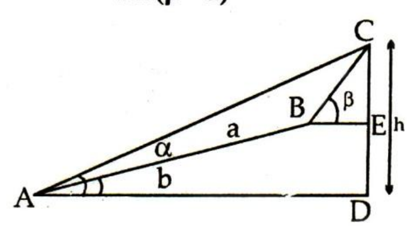

**6.** The angle of elevation of a cloud from a height 'd' metres above the level of water in a lake is 'a' and the angle of depression of its image in the lake is

$$h = \frac{d\sin(\beta+\alpha)}{\sin(\beta-\alpha)} \textit{(or)}\ h = \left[\frac{d(\tan\beta+\tan\alpha)}{(\tan\beta-\tan\alpha)}\right] \quad \text{or, } h = d\left[\frac{\text{Cot}\,\alpha + \text{Cot}\,\beta}{\text{Cot}\,\alpha - \text{Cot}\,\beta}\right]$$



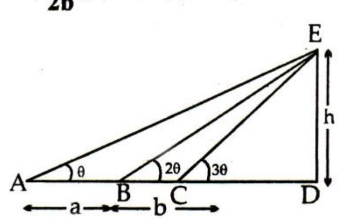

**7.** The angle of elevation of hill from a point A is 'a' . After walking to some point B at a distance 'a' metres from A on a slope inclined at '$\gamma$' to the horizon, the angle of elevation was found to be $\beta$

$$\text{Height of the hill } h = \frac{a \text{ Sin}\,\alpha \text{ Sin}(\beta-\gamma)}{\text{Sin}(\beta-\alpha)}$$

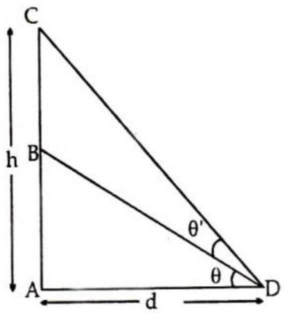



**8.** A ballon is observed simultaneously from the three points A, B, C on a straight road directly beneath it. The angular elevation at B is twice that at A and the angular elevation at 'C' is thrice that at A. If AB = a and BC = b then the height of the ballon h in terms of a and b is,

$$h = \frac{a}{2b}\sqrt{(3b-a)(a+b)}$$



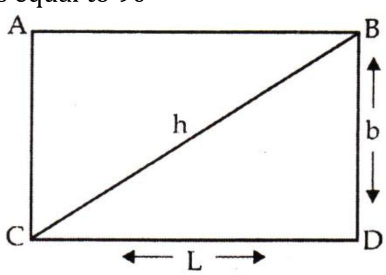
**9.** A flag staff stands on the top of a tower of height h metres. If the tower and flag staff subtend equal angles at a distance 'd' metres from the foot of the tower, then the height the flag-staff in metres is $h\left[\frac{d^2+h^2}{d^2-h^2}\right]$




# MENSURATION

## 1. Rectangle

A Quadrilateral with opposite sides equal and all the four angles equal to 90°



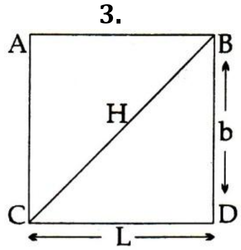

- Perimeter of Rectangle - $2(l+b)$
- Area of Rectangle - $(l \times b)$
- Other formula based on Rectangle

$h=\sqrt{l^2+b^2}$, where L (length)
B (Breath)
H (diagonal)

- Length $= \dfrac{Area}{Breadth}$; $Breadth = \dfrac{Area}{Length}$
- $(Diagonal)^2 = (Length)^2 + (Breadth)^2$

## 2. Square

3.

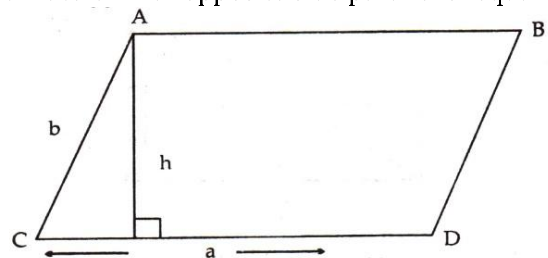



- Perimeter of Square : $4 x l$
- Area of Square : $(side)^2 / l^2 = \frac{1}{2}h^2$
- Others Formula : $h=\sqrt{l^2+l^2}$

$= \sqrt{2l^2} = \sqrt{2 x\, area}$

## 3. Parallelogram

A quadrilateral with opposite side parallel of equal.

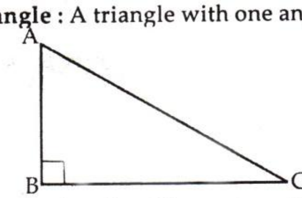



- Perimeter of Parallelogram : **2(a+b)**
- Area of Parallelogram : **a x h**
- Others : $\mathbf{base = \frac{area}{height}}$

## 4. Triangle

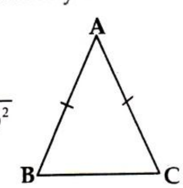
Three types of Triangle

i) **Right angled triangle :** A triangle with one angle equal to $90^\circ$



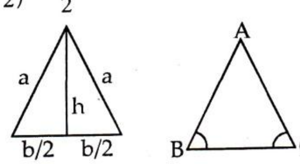

ii) **Isoceles triangle :** A triangle with any two sides equal

$Area = \frac{b}{4}\sqrt{4a^2-b^2}$



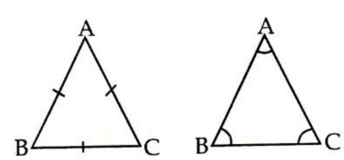

**In an isoceles triangle, opposite angles are equal**

$\Rightarrow h=\sqrt{a^2-\left(\frac{b}{2}\right)^2} = \frac{1}{2}\sqrt{4a^2-b^2}$



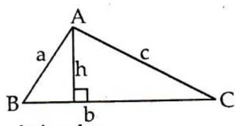

$\angle B = \angle C$

iii) **Equilateral triangle :** A triangle with all sides equal and all angles are equal.



- $Area = \frac{\sqrt{3}}{4} x (side)^2$
- Perimeter of an Equilateral triangle : 3 x Side

**iv) Scalene triangle(Heroine Formulas) :** A triangle with no any side equal




- **Perimeter of triangle :**

  $2S = a+b+c$

  $S = \frac{a+b+c}{2}$

- **Area of triangle :** $\sqrt{s(s-a)(s-b)(s-c)}$

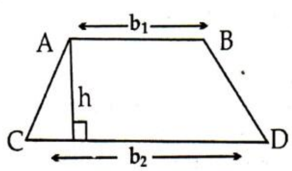
  $= : \frac{1}{2} \times \textbf{base} \times \textbf{height}$

- Others : $\mathbf{h} = \frac{\sqrt{3}}{2} \times \textbf{side}, \therefore \textbf{side} = \frac{2\mathbf{h}}{\sqrt{3}}$

## 5. Trapezium

A quadrilateral with any one pair of opposite sides parallel.

⇒ **Area of trapezium**

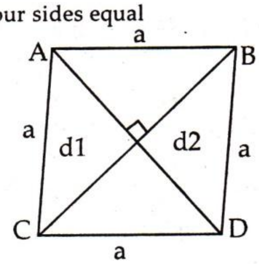



= $\frac{1}{2}$ (sum of parallel sides) x perpendicular distance between them

$$\frac{1}{2} (b_1 + b_2) h$$

## 6. Rhombus

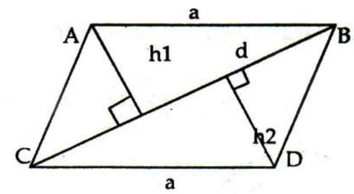
A Parallelogram with all four sides equal




- **Area of a Rhombus :** $\frac{1}{2}$ x (Product of diagonal)

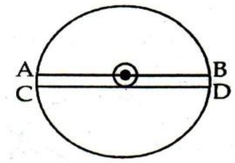
or Area = **Breadth × Height**

## 7. Quadrilateral




- **Area of quadrilateral :** $\frac{1}{2} \times d \times (h_1 + h_2)$

## 8. Circle




- **Perimeter of Circle :** $2\pi r \; or \; \pi D (d = 2r)$
- Perimeter of Semi Circle = $\pi r + 2r$
- Area of Circle : $\pi r^2$
- Area of Semi Circle = $\frac{1}{2} \pi r^2$

Others – Circumference = $2\pi r \; or \; \pi D$

$\therefore$ D = 2r

r= $\frac{D}{2}$, where r = radius OA = OB

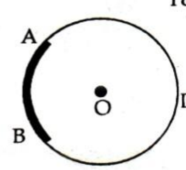
D = diameter = AB

CD = Chord

## 9. i) ARC of a Circle

A portion of the Perimeter (or a part of the curved portion) of the circle.

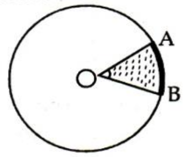
- Length of Arc $l = \frac{\theta}{180} \times \pi r \; or, \frac{\theta}{360} \times 2\pi r$




ARC AB=Length of AB

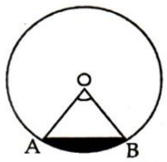
- Arc AB = $\frac{2\pi r\theta}{360°}$, $where \angle AOB = \theta \; and \; O \; is \; the \; center$

ii) **Sector of a circle:** The area covered between an arc, the centre and two radii of the circle.
shaded portion = sector AOB




- Area of sector AOB = $\frac{\pi r^2\theta}{360°}$
- Area of Sector AOB = $\frac{1}{2} \times arc\ AB \times r$

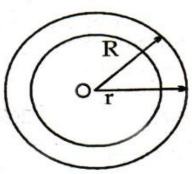
iii) **Segment of a Circle :**




- Area of Segment of a circle = $r^2\left[\frac{\pi\theta}{360°} - \frac{\sin\theta}{2}\right]$

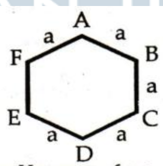
or, $\frac{\pi r^2\theta}{360°} - \frac{1}{2}r^2 \sin\theta$

(iv) **Ring :** $\pi\ (R^2 - r^2)$




R = Radius of bigger circle
r = Radius of smaller circle

**10. Regular Hexagon**



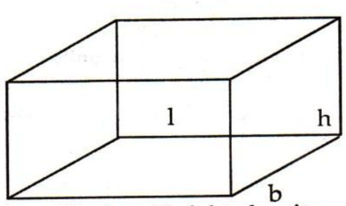

- Perimeter of Regular Hexagon : 6 x a
- Area of Regular Hexagon : $\frac{3\sqrt{3}}{2} \times a^2$
- Others : Each interior angle
- $\frac{(2n-4)}{n} \times 90^0$

Sum of total interior angles = $(2n - 4) \times 90°$

**12. Area of 4 walls of a room = $2 \times (l + b) \times h$ or Perimeter × Height**

## Mensuration – II

**1. Cuboid :**




**Let length = L, breadth = b and height = h units**

i. Volume of Cuboid = (l x b x h) Cubic Units

$= \sqrt{A_1 \times A_2 \times A_3}$ cm. Units

Where $A_1$ = area of base or top
$A_2$ = area of side face
$A_3$ = area of other side face

ii) Whole surface area of cuboid = $2(lb + bh + lh)$ sq. Units

iii) Diagonal of Cuboid = $\sqrt{l^2 + b^2 + h^2}$ Units

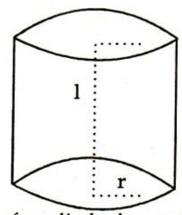
**2. Cube :**
**Let each edge (or side) of a cube be a units. Then**

i) Volume of the cube = $a^3$ cubic units

ii) Whole surface area of the cube = $(6a^2)$ sq. units

iii) Diagonal of the cube = $\left(\sqrt{3}a\right)$ units.

**3. Cylinder**




Let the radius of the base of a cylinder be r units and its height (or length) be h units Then:

i) Volume of the cylinder = $(\pi r^2 h)$ cu. units

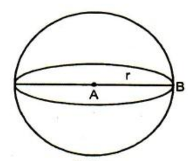
ii) curved surface area of the cylinder = $(2\pi rh)$ sq. units

iii) Total surface area of the cylinder = $(2\pi rh + 2\pi r^2)$ sq. units

**4. Sphere :** Let the radius of a sphere be r units Then :

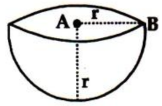
i) Volume of the sphere = $\left(\frac{4}{3}\pi r^3\right)$ cu. units

ii) Suface area of the sphere = $\left(4\pi r^2\right)$ sq. units




**Sphere**

**5. Hemisphere:**

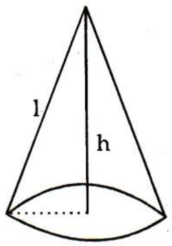



i) Volume of a hemisphere = $\left(\frac{2}{3}\pi r^3\right)$ cu. units

ii) Curved surface area of the hemisphere = $\left(2\pi r^2\right)$ sq. units

iii) Whole surface area of the hemisphere = $\left(3\pi r^2\right)$ sq. units

**6. Right Circular Cone :-**




Let r be the radius of the base, h the height and $l$ the slant height of a cone. Then:

i) Slant height $l = \sqrt{h^2 + r^2}$

ii) Volume of the cone = $\left(\frac{1}{3}\pi r^2 h\right)$ cu. units

iii) Curved Surface area of the cone :

$= \pi r l \; sq.units$

iv) Total surface area of the cone

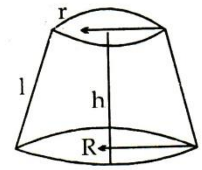
$= (\pi r l + \pi r^2)$

$= \pi r (l + r)$ or

**7. Frustum of a right circular cone :-** If a cone is cut by a plane parallel to the base so as to divide the cone into two parts. The lower part is called **frustum of the cone.**

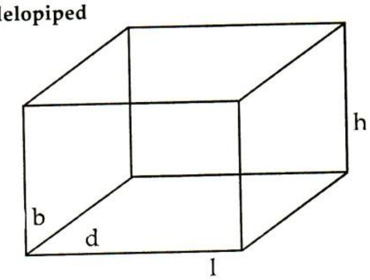
Let the radius of the base of the frustum = R, the radius of the top = r, height = h & slant height = $l$ units






- Curved surface area = $\pi(r+R)l \; sq.units$
- Total surface area = $\pi\{(r+R)l + r^2 + R^2\} sq.units$
- Volume = $\frac{\pi h}{3}(r^2 + R^2 + rR) cu.units$

**8. Right Parallelopiped**




- Surface area (of the sides faces) = $2h(b+l) sq.units$
- Surface area (of the base or the top face) $= 2\sqrt{s(s-a)(s-b)(s-d)}\, sq.units$
- Total surface area = $2h(b+l) + 4\sqrt{s(s-a)(s-b)(s-d)}\, sq.units$
- Volume = Base area x height

## CIRCLE

**Definition** : A circle is the locus of a point which moves so that its distance from a fix point.

i. Equation of a circle
   a. $(x - h)^2 + (y - k)^2 = a^2$
   b. $x^2 + y^2 = a^2$ for the origin

ii. General Equation : If centre is (h, k) and radius a is

$$(x - h)^2 + (y - k)^2 = a^2$$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

**where g = - h, f = -k and c = $h^2 + k^2$ - a**

$\therefore$, Centre is **(-g, -f)** and radius is $\sqrt{g^2 + f^2 - c}$

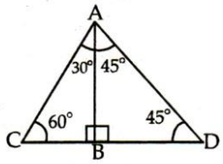
iii. Conditions for the equation of a circle for the general from.

**$x^2 + y^2 + 2gx + 2fy + c = 0$ multiply throughout by a (i.e.)**

$$ax^2 + ay^2 + 2gax + 2fay + ac = 0$$

a. It should be an equation of the second degree in x and y.
b. The co-efficients of $x^2$ and $y^2$ should be equal.
c. There should be no term involving the product xy.
d. Diameter from $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

1. Theorem of $30° - 60° - 90°$ triangle : If the angles of a triangle are $30°, 60°$ *and* $90°$, then the side opposite to $30°$ is half of the hypotenuse & the side opposite to $60°$ is $\frac{\sqrt{3}}{2}$ times the hypotenuse.




i.e. $BC = \frac{1}{2}AC$, $AB = \frac{\sqrt{3}}{2}AC^2$

2. Theorem $45° - 45° - 90°$ Triangle. if the angles of triangle are $45°, 45°, 90°$ then the perpendicular side are $\frac{1}{\sqrt{2}}$ times the hypotenuse e.g. in the above figure.

$AB = BD = \frac{1}{\sqrt{2}} AD$

# ALL The Best for upcoming examinations !!