Electric Potential Difference – It is measure of work done by external agent to move a test charge between two points.

$$V_A - V_B = \frac{W_{ext}(B \to A)}{q_0}$$

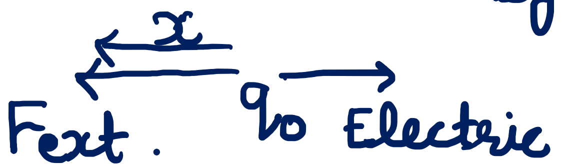
and test charge must be extremely small and positive.

* The charge q_0 should be moved very slowly so that the kinetic energy do not change (V = constant, $a = 0$).

* Unit of potential difference is – Joule/Coulomb or Volt.

\# Potential at a point is defined as the amount of work done (by external agent) per unit charge in bringing the charge from infinite (∞) to that point without any change in its kinetic energy.

→ Work done by external agent against electrostatic force is

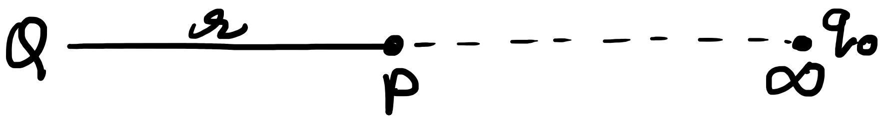


$$\vec{F}_{ext} = -F_{electric} \Rightarrow W_{ext\ agent} = -W_{electric\ force}$$

→ Work done by any conservative force is path independent (depends only on initial and final position)

$$\boxed{W_1 = W_2 = W_3}$$

Electric Potential due to a Point Charge (at distance 'r' from it) –



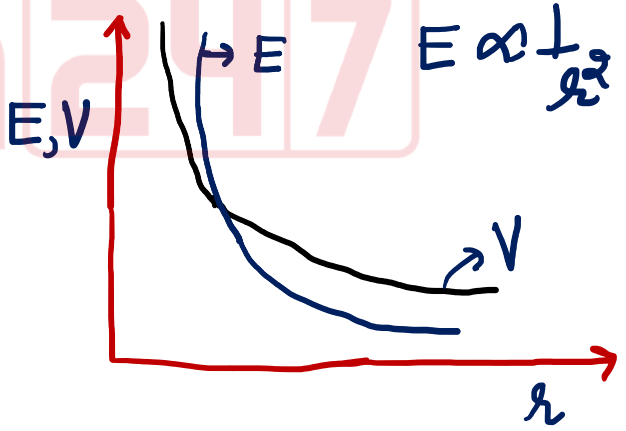
$$V_p = \frac{W_{ext}(\infty \to P)}{q_0} \Rightarrow V_p = \frac{KQ}{r}$$

* The influence of a charge in its surroundings can be described in two ways

i) Electric field, $E_p = \frac{KQ}{r^2}$ (vector)

ii) Electric Potential, $V_p = \frac{KQ}{r}$ (scalar)   Q with sign.

$$E = \frac{KQ}{r^2}$$



$E \propto \frac{1}{r^2}$, $V \propto \frac{1}{r}$

Q1. i) Calculate the potential at a point P due to a charge of $4\times10^{-7}C$ located 9 cm away.

ii) Hence obtain the work done in brining a charge of $2\times10^{-9}C$ from infinity to the point P. Does the answer depend on path along which charge is brought?

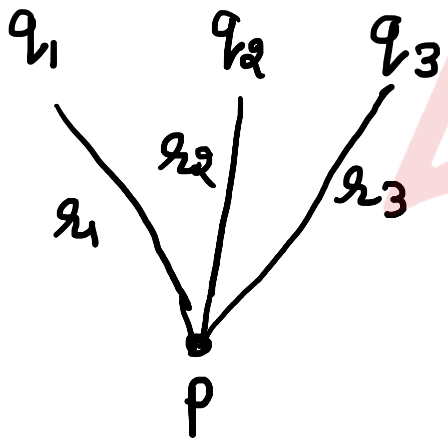
Sol. Q ———— 9cm ———— P

$Q = 4\times10^{-7}C$

$$V_p = \frac{kQ}{r} = \frac{9\times10^{9}\times4\times10^{-7}}{9\times10^{-2}} = 4\times10^{4}\,V$$

$$W_{ext\,(\infty\to P)} = V_p\times q = 4\times10^{4}\times2\times10^{-9} = 8\times10^{-5}\,J$$

Electric Potential due to a System of Charges –



$$V_p = V_1+V_2+V_3$$

$$= \frac{kq_1}{r_1}+\frac{kq_2}{r_2}+\frac{kq_3}{r_3} \quad \text{(scalar addition)}$$

$V = \frac{kQ}{r} \rightarrow$ Point charge.

Q2. Three charges $q_1 = 1\mu C$, $q_2 = -2\mu C$, $q_3 = -1\mu C$ are placed at A(0,0,0), B(-1,2,3) and C(2,-1,1). Find potential of the system of three charges at P(1,-2,-1).

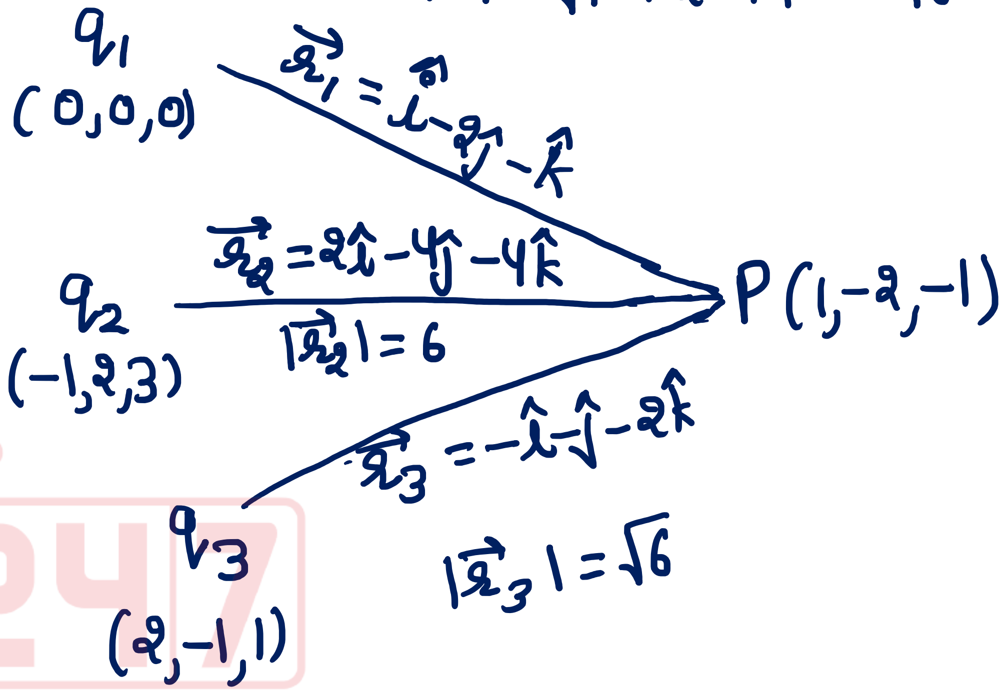
Sol.

$$V_1 = \frac{kq_1}{r_1} = \frac{9\times10^9 \times 1\times10^{-6}}{\sqrt{6}} = \frac{9\times10^3}{\sqrt{6}}\,V$$

$$V_2 = \frac{kq_2}{r_2} = \frac{9\times10^9 \times (-2)\times10^{-6}}{6} = -3\times10^3\,V$$

$$V_3 = \frac{kq_3}{r_3} = \frac{9\times10^9 \times (-1)\times10^{-6}}{\sqrt{6}} = \frac{-9\times10^3}{\sqrt{6}}$$

$$|r_1| = \sqrt{1^2+2^2+1^2} = \sqrt{6}$$



Q3. Calculate the electric potential at the centre of a square of side $\sqrt{2}$ m, having charge $100\mu C$, $-50\mu C$, $20\mu C$ and $-60\mu C$ at the four corner of the square.

Sol.

$$V_P = V_1 + V_2 + V_3 + V_4 = \frac{kq_1}{r} + \frac{kq_2}{r} + \frac{kq_3}{r} + \frac{kq_4}{r}$$

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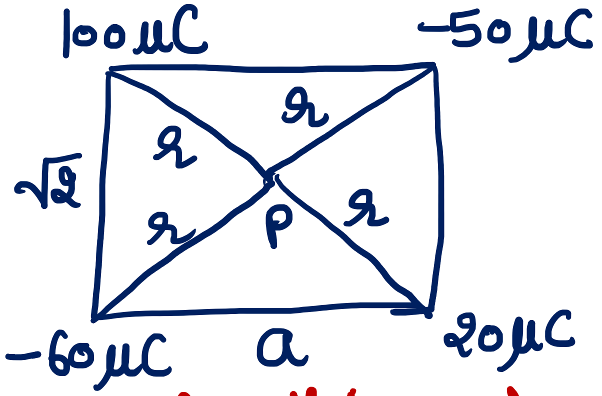
$$V_p = \frac{k\times 10^{-6}}{r}\left(100+(-50)+20+(-60)\right) = \frac{9\times 10^{9}\times 10^{-6}}{1}(10) = 9\times 10^{4}\,V$$

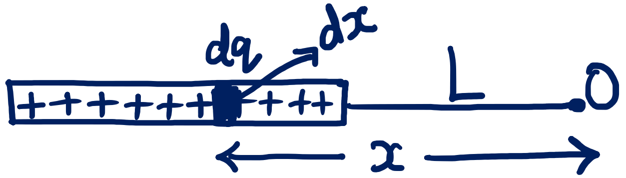
$$\left[\because \quad r = \frac{\text{diagonal}}{2} \Rightarrow r = \frac{a\sqrt{2}}{2} = \frac{\sqrt{2}\times\sqrt{2}}{2} = 1\,m\right]$$

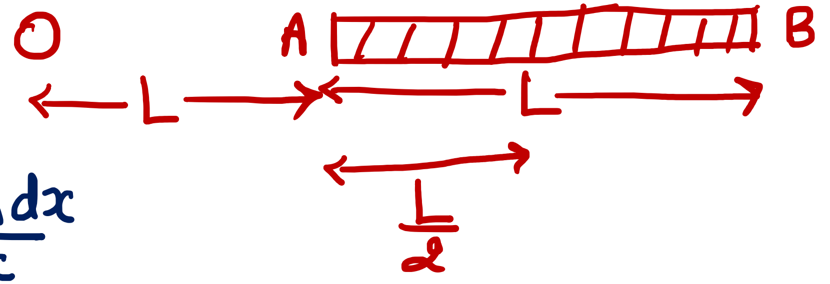


Q4. A charge Q is uniformly distributed over a rod AB of length L as shown in figure. The electric potential at the point O lying at distance L from the end A is -

a) $\frac{Q}{8\pi\epsilon_0 L}$ b) $\frac{3Q}{4\pi\epsilon_0 L}$ c) $\frac{Q}{4\pi\epsilon_0 L \ln 2}$ d) $\frac{Q\ln 2}{4\pi\epsilon_0 L}$

Sol.



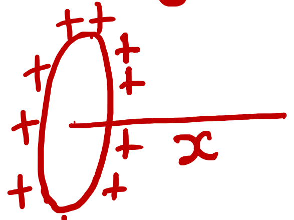


Potential due to dq, $V = \int dV = \int \frac{k\,dq}{x} = \int_{L}^{2L} \frac{k\lambda\,dx}{x}$

$$V = k\lambda \int_{L}^{2L} \frac{dx}{x} = k\lambda \left[\log_e x\right]_{L}^{2L} = k\lambda\left[\log_e(2L) - \log_e(L)\right] = k\lambda \log_e\left(\frac{2L}{L}\right)$$

$$\Rightarrow \quad V = k\lambda \log_e 2 = \frac{1}{4\pi\epsilon_0}\frac{Q}{L}\log_e 2 = \frac{Q}{4\pi\epsilon_0 L}\log_e 2$$

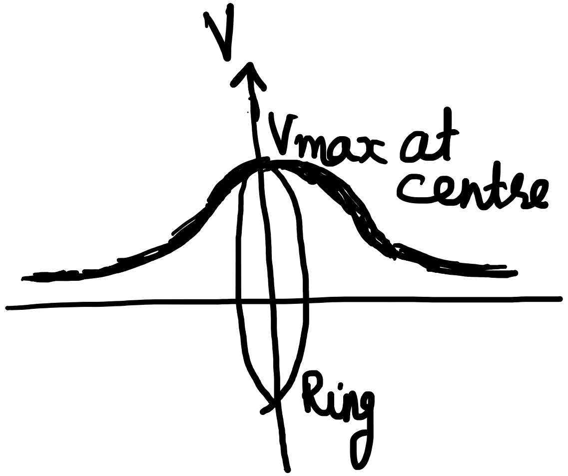
Q5. Find electric potential on the axis of a ring of radius 'R' having charge 'Q' uniformly distributed on it.



Sol. $dV = \frac{k\,dq}{r}$

$dV = \int \frac{k\,dq}{\sqrt{R^2+x^2}}$

$$V = \int dV = \int \frac{k\,dq}{\sqrt{R^2+x^2}} = \frac{k}{\sqrt{R^2+x^2}}\int_{Ring} dq \Rightarrow V_{axis} = \frac{kQ}{\sqrt{R^2+x^2}}$$

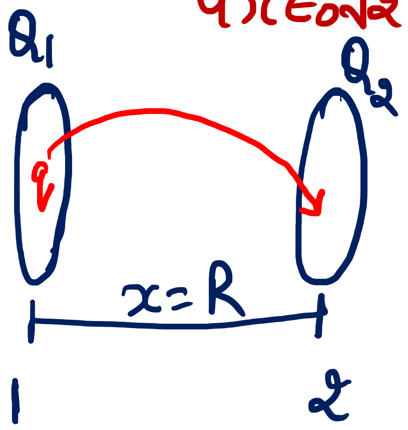


Centre, $x = 0$, $\quad V_{centre} = \frac{kQ}{R}$ (max)

Q.6. Two identical thin rings each of radius R are co-axially placed at a distance R apart. If Q_1 and Q_2 are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to the other is :

a) $\dfrac{q\left(\frac{Q_1}{Q_2}\right)(\sqrt{2}+1)}{4\pi\epsilon_0\sqrt{2}\,r}$ b) $\dfrac{q(Q_1-Q_2)(\sqrt{2}-1)}{4\pi\epsilon_0\sqrt{2}\,R}$ c) $\dfrac{q\sqrt{2}\,(Q_1+Q_2)}{4\pi\epsilon_0 R}$ d) zero

Sol.



$$V_2 - V_1 = \frac{W_{ext}(1-2)}{q}$$

$$V_1 = \underset{(\text{centre})}{V_{Q_1}} + \underset{(\text{axis})}{V_{Q_2}} = \frac{kQ_1}{R} + \frac{kQ_2}{\sqrt{R^2+R^2}} = \frac{kQ_1}{R} + \frac{kQ_2}{R\sqrt{2}}$$

$$V_2 = \underset{(\text{centre})}{V_{Q_2}} + \underset{(\text{axis})}{V_{Q_1}} = \frac{kQ_2}{R} + \frac{kQ_1}{\sqrt{2}R}$$

$$V_2 - V_1 = \frac{kQ_2}{R} + \frac{kQ_1}{\sqrt{2}R} - \frac{kQ_1}{R} - \frac{kQ_2}{\sqrt{2}R} = \frac{k}{R}\left(1-\frac{1}{\sqrt{2}}\right)(Q_2-Q_1) = \frac{1}{4\pi\epsilon_0 R}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)(Q_2-Q_1)$$

$$\Rightarrow W_{ext} = q(V_2-V_1) = q \times \frac{1}{4\pi\epsilon_0} \times \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)(Q_2-Q_1)$$

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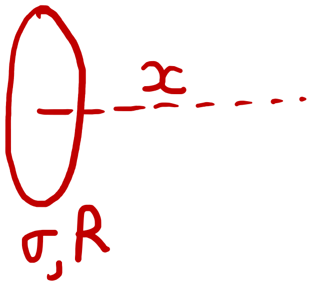
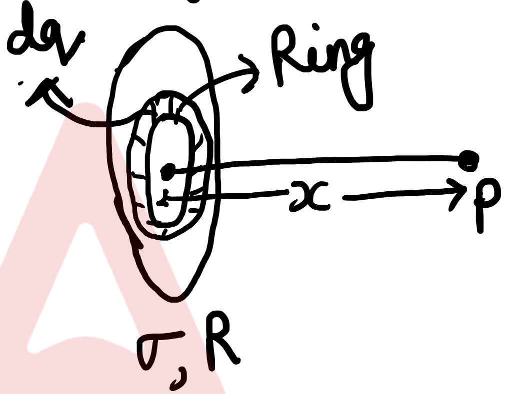
[Electric Potential on the Axis of a Uniformly Charged Disc]

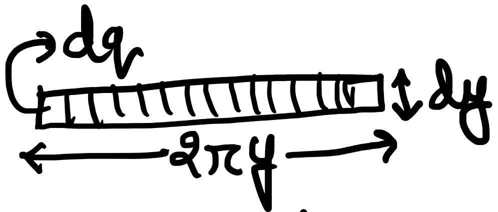


elemental charge $dq \longrightarrow$ Ring

Potential dV due to elemental ring (radius $= y$)

$$dV = \frac{k\,dq}{\sqrt{y^2+x^2}} \Rightarrow V = \int dV = \int_{y=0}^{y=R} \frac{k\,dq}{\sqrt{y^2+x^2}}$$





$dA = 2\pi y\,dy$

$\sigma = \frac{dq}{dA} \Rightarrow dq = \sigma\,dA$

$$\therefore V = k\int_{y=0}^{y=R} \frac{\sigma\,dA}{\sqrt{y^2+x^2}} = k\int_{y=0}^{y=R} \frac{\sigma(2\pi y)\,dy}{\sqrt{y^2+x^2}}$$

$$\therefore V = k\sigma 2\pi \int_0^R \frac{y\,dy}{\sqrt{y^2+x^2}}$$

Let $y^2+x^2 = t^2 \Rightarrow 2y\,dy + 0 = 2t\,dt \Rightarrow y\,dy = t\,dt \Rightarrow V = k\sigma 2\pi \int \frac{t\,dt}{\sqrt{t^2}}$

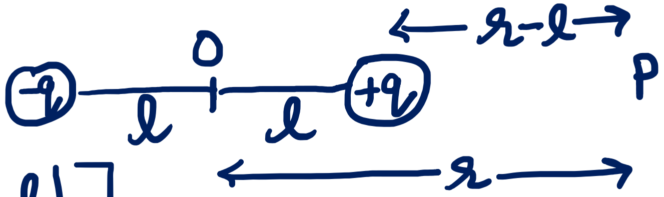
$$\Rightarrow V = k\sigma 2\pi \int dt = k\sigma 2\pi \left[\sqrt{y^2+x^2}\right]_0^R = \frac{1}{4\pi\epsilon_0}\sigma 2\pi\left(\sqrt{R^2+x^2} - x\right) \Rightarrow V_{\text{axis (disc)}} = \frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+x^2} - x\right)$$

$$V_{\text{axis (disc)}} = \frac{\sigma}{2\epsilon_0}\left[\sqrt{R^2+x^2} - x\right] \Rightarrow \text{At Centre } (x=0),\ \boxed{V = \frac{\sigma R}{2\epsilon_0}}$$

[Electric Potential Due to a Dipole]

A. On the axis of a dipole at distance r from centre of dipole -

$$V_1 = \frac{kq}{r-l} \qquad V_2 = \frac{-kq}{r+l} \quad \Rightarrow V = V_1 + V_2$$



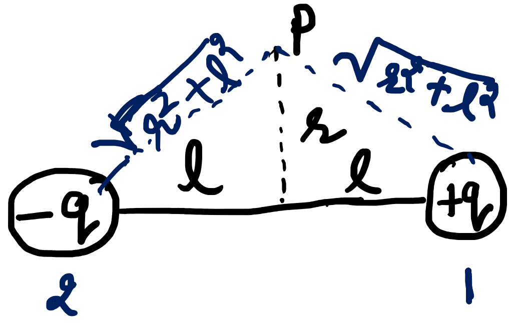
$$V = \frac{kq}{r-l} - \frac{kq}{r+l} = kq\left(\frac{1}{r-l} - \frac{1}{r+l}\right) = kq\left[\frac{(r+l)-(r-l)}{(r-l)(r+l)}\right]$$

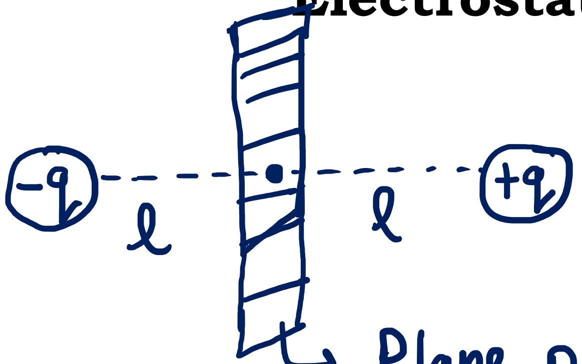
$$= kq\left(\frac{2l}{r^2-l^2}\right) \Rightarrow V = \frac{kp}{r^2-l^2} \text{ at axis}$$

For far off distance $l <<< r$ or $l^2 \longrightarrow 0$ $\Rightarrow \boxed{V_{axis} = \frac{kp}{r^2}}$

B. On the equitorial line of Dipole (perpendicular bisector of dipole) (at distance r from centre of dipole) -

$$V_1 = \frac{kq}{\sqrt{r^2+l^2}} \,, \quad V_2 = \frac{-kq}{\sqrt{r^2+l^2}} \Rightarrow V_p = V_1 + V_2 = 0$$



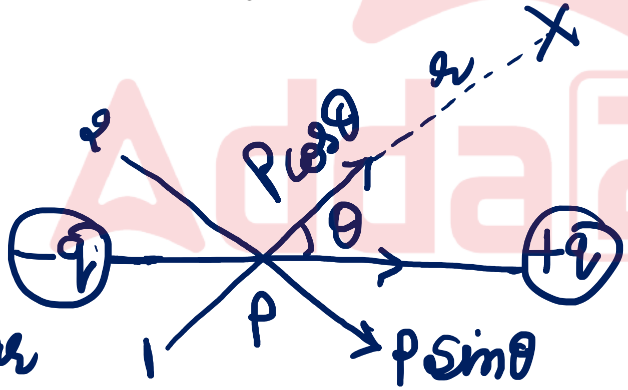


Plane perpendicular to line joining +q & -q and midway between them, $V=0$ (equitorial surface)

c. At some general point (r,θ) due to short dipole $(2l \to 0)$

Two dipole –

$P\cos\theta$	$P\sin\theta$
↓ X is on axis	X is on perpendicular bisector



$$V_x = \frac{KP\cos\theta}{r^2} + 0 \Rightarrow V_x = \frac{KP\cos\theta}{r^2}$$

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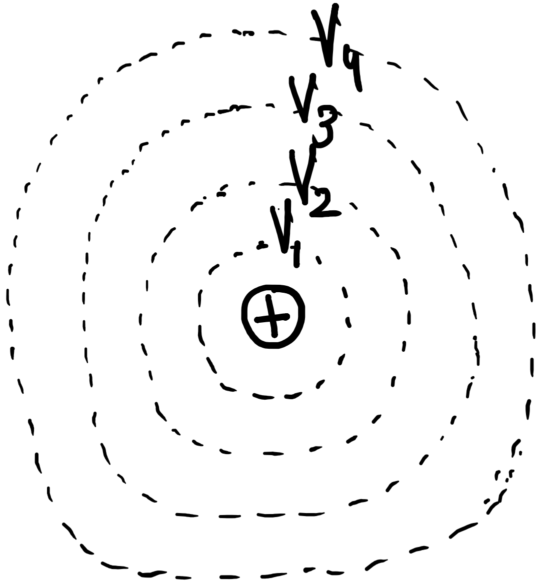
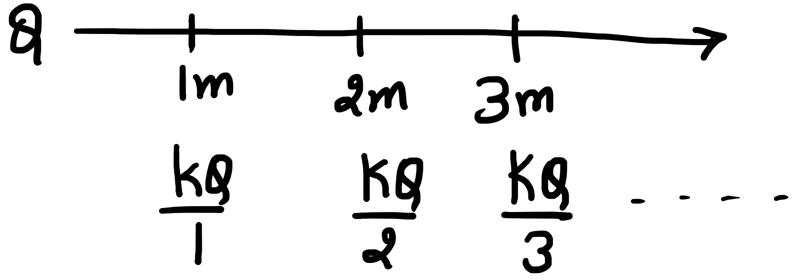
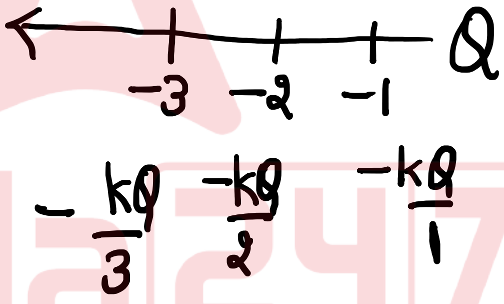
Relation between Electric field ($\vec{E}$) and Electric Potential (V)

* Electric Potential always decrease in the direction of electric field.



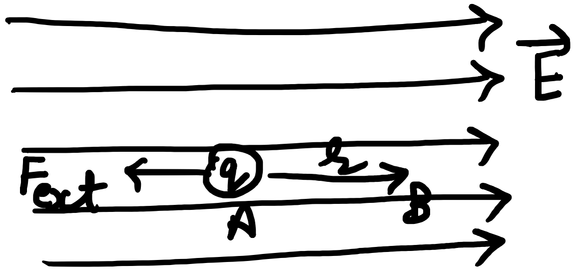
$V_1 > V_2 > V_3 > V_4$





Relation between $\vec{E}$ and ΔV

Case - 1 : If $\vec{E}$ is uniform



$F_{ext} = -F_{electric} = -qE$

$V_B - V_A = \frac{W_{ext}(A \to B)}{q} = \frac{F_{ext} \cdot r}{q} = \frac{-q\vec{E} \cdot \vec{r}}{q} = -\vec{E} \cdot \vec{r}$

$\Delta V = -\vec{E} \cdot \vec{r} \Rightarrow |\Delta V| = \vec{E} \cdot \vec{r}$ or $\vec{E} = \frac{\Delta V}{r}$

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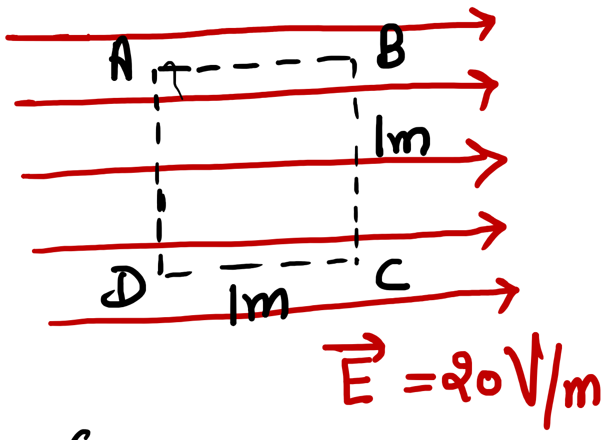
Q7. In the uniform electric field shown in figures, Find:



a) $V_C - V_D$

b) $V_A - V_D$

c) $V_B - V_D$

d) $V_A - V_C$

a) $|\Delta V| = \vec{E}\cdot\vec{r} = Er\cos 0^\circ = 20\times1\times1 = 20V$

b) $|\Delta V| = Er\cos\theta = Er\cos 90^\circ = 0$

c) $|\Delta V| = Er\cos 45^\circ = 20\times\sqrt{2}\times\frac{1}{\sqrt{2}} = 20V$

d) $|\Delta V| = Er\cos 45^\circ = 20\times\sqrt{2}\times\frac{1}{\sqrt{2}} = 20V$

Sol. a) $V_C - V_D = -20V$

b) $V_A - V_D = 0$

c) $V_B - V_D = -20V$

d) $V_A - V_C = 20V$

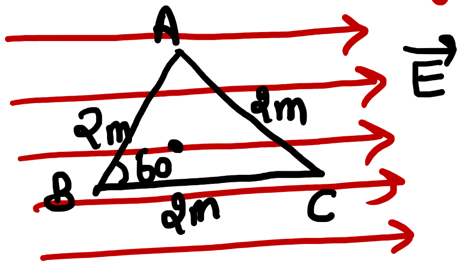
Potential always decrease in the direction of electric field.

$V_C < V_D$

$V_B < V_D$

$V_C < V_A$

Q8. In uniform electric field $\vec{E} = 10\,N/C$ as shown in figure, Find



a) $V_A - V_B$

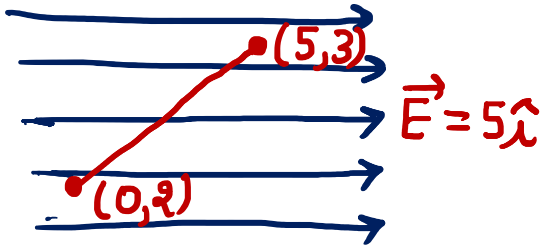
b) $V_B - V_C$

a) $V_A - V_B$ $\Rightarrow |\Delta V| = \vec{E}.\vec{r} = Er\cos 60^\circ = 10 \times 2 \times \frac{1}{2} = 10V$ or $V_A - V_B = -10V$

b) $V_B - V_C$ $\Rightarrow |\Delta V| = \vec{E}.\vec{r} = Er\cos 0^\circ = 10 \times 2 \times 1 = 20V$ or $V_B - V_C = 20V$

Q9. A uniform electric field is present in the positive x-direction. If the intensity of field is 5N/C. Then find the potential difference $(V_A - V_B)$ between two points A (0m, 2m) and B(5m, 3m).

a) $-25V$ b) $-25V$ c) $75V$ d) $-75V$



Sol. $|\Delta V| = \vec{E}.\vec{r} = (5\hat{i})(5\hat{i}+\hat{j}) = 25V$

$V_B - V_A = -25V$

or A (0,2) ———— B (5,3)

$\vec{r} = 5\hat{i} + \hat{j}$

$V_B - V_A = -\vec{E}.\vec{r} = -(5\hat{i})(5\hat{i}+\hat{j}) = -25V$

Q10. An electric field is expressed as $\vec{E} = (2\hat{i} + 3\hat{j})\,V/m$. Find the potential difference $(V_A - V_B)$ between two points A and B whose position vectors are given by $\vec{r}_A = (\hat{i} + 2\hat{j})$ and $\vec{r}_B = (2\hat{i} + \hat{j} + 3\hat{k})\,m$.

a) -1 b) 1 c) -2 d) 2

Sol. $\vec{E} = 2\hat{i} + 3\hat{j}$ $B \xrightarrow{\vec{r}} A$ $\vec{r} = -\hat{i} + \hat{j} - 3\hat{k}$

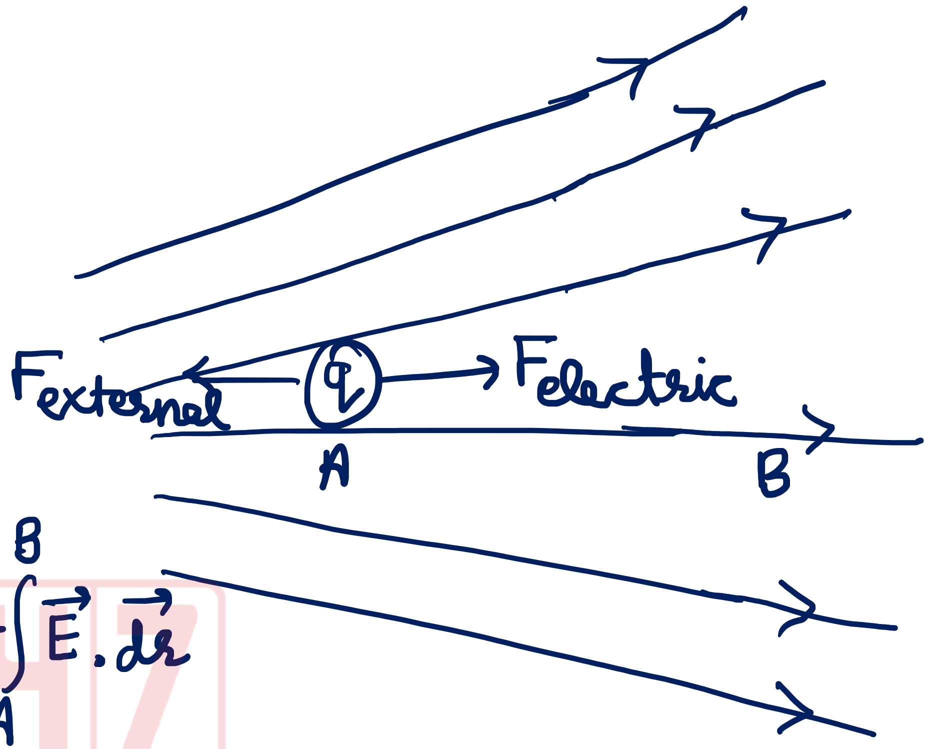
$$V_A - V_B = -\vec{E}\cdot\vec{r} = -(2\hat{i}+3\hat{j})\cdot(-\hat{i}+\hat{j}-3\hat{k}) = -[2\times(-1) + 3\times(1) + 0\times -3]$$

$$= -[-2+3] = -1V$$

Case-II : If $\vec{E}$ is non-uniform –

$$F_{ext} = -F_{electric} = -q\vec{E}$$

$$dV = \frac{dW_{ext}}{q} = \frac{F_{ext}\cdot dr}{q} = \frac{-q\vec{E}\cdot dr}{q}$$



$$dV = -\vec{E}\cdot\vec{dr} \Rightarrow V_B - V_A = \Delta V = \int dV = -\int_A^B \vec{E}\cdot\vec{dr}$$

uniform electric field

$$\Delta V = -\vec{E}\cdot\vec{r}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

non-uniform $\vec{E}$

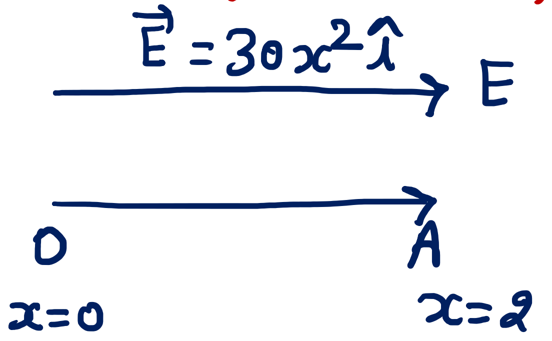
$$\Delta V = -\int \vec{E}\cdot\vec{dr}$$

$$\vec{dr} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

Q11. $\vec{E} = 30x^2\hat{i}$. Find $V_A - V_0$, V_A is potential at $x = 2m$, V_0 is potential at origin.

a) 120 V b) −120 V c) −80 V d) 80 V

Sol.



$$V_A - V_0 = -\int_O^A \vec{E}\cdot\vec{dr} = -\int_O^A (30x^2\hat{i})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= -\int_0^2 30x^2dx = -30\int_0^2 x^2dx = -30\left[\frac{x^3}{3}\right]_0^2$$

$$V_A - V_0 = -10\left[x^3\right]_0^2 = -80V$$

Q12. $E = (20x + 10)\hat{i}$. If potential at $x = 1$ is V_1 and $x = -5$ is V_2. Find $V_1 - V_2$.

a) −48 V b) −520 V c) 180 V d) 320 V

Sol. $\vec{E} = (20x + 10)\hat{i}$

$$V_1 - V_2 = -\int_2^1 \vec{E}\cdot\vec{dr}$$

$$V_2 - V_1 = -\int_2^1 (20x + 10)\hat{i}\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k}) = -\int_{-5}^1 (20x+10)dx$$

$$V_2 - V_1 = -\int_{-5}^{1}(20x+10)\,dx = -\left[\frac{20x^2}{2}+10x\right]_{-5}^{1} = -\left[10x^2+10x\right]_{-5}^{1} = -10\times -18$$

$$\therefore V_2 - V_1 = 180\,V$$

Q13. $\vec{E} = (x\hat{i} - 2y\hat{j} + z\hat{k})\ V/m$. Find $V_B - V_A$ if $A(0,2,4)$ and $B(2,1,0)$.

a) 9V b) 3V c) −9V d) −3V

Sol. $\vec{E} = x\hat{i} - 2y\hat{j} + z\hat{k}$

A (0,2,4) —— B (2,1,0)

$$V_B - V_A = -\int_A^B \vec{E}\cdot\vec{dr} = -\int_A^B (x\hat{i} - 2y\hat{j} + z\hat{k})\cdot(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

$$= -\int_{(0,2,4)}^{(2,1,0)} x\,dx - 2y\,dy + z\,dz$$

$$= -\left[\frac{x^2}{2} - \frac{2y^2}{2} + \frac{z^2}{2}\right]_{0,2,4}^{2,1,0} = -\frac{1}{2}[2-8]$$

$$= 3V$$

[Finding Electric field from expression of V]

$dV = -\vec{E}\cdot\vec{dr} = -(E_x\hat{i} + E_y\hat{j} + E_z\hat{k})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k}) = -E_x dx - E_y dy - E_z dz$

or $\vec{E} = -$ Potential gradient

Suppose only x is changing, y and z be have as constant.

$dy = 0$, $dz = 0$

$dV = -E_x dx \Rightarrow \boxed{E_x = -\frac{dV}{dx}}$ → partially correct

and correct relation, $E_x = -\frac{\partial V}{\partial x}$

$\boxed{E_y = -\frac{\partial V}{\partial y}}$ and $\boxed{E_z = -\frac{\partial V}{\partial z}}$

Q 14. $V = \frac{x^2}{2} - \frac{y^3}{3} + z$. Find $\vec{E}$ at (1,2,0) and also calculation $|\vec{E}|$ at (1,2,0).

Sol. $V = \frac{x^2}{2} - \frac{y^3}{3} + z$, $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k} = -x\hat{i} + y^2\hat{j} - \hat{k} \Rightarrow \vec{E}_{(1,2,0)} = -\hat{i} + 4\hat{j} - \hat{k}$

$E_x = -\frac{\partial V}{\partial x} = -\left[\frac{2x}{2} + 0 + 0\right] = -x$ | $E_y = -\frac{\partial V}{\partial y} = -\left[0 - \frac{3y^2}{3} + 0\right] = y^2$ | $E_z = -\frac{\partial V}{\partial z} = -[0+0+1] = -1$

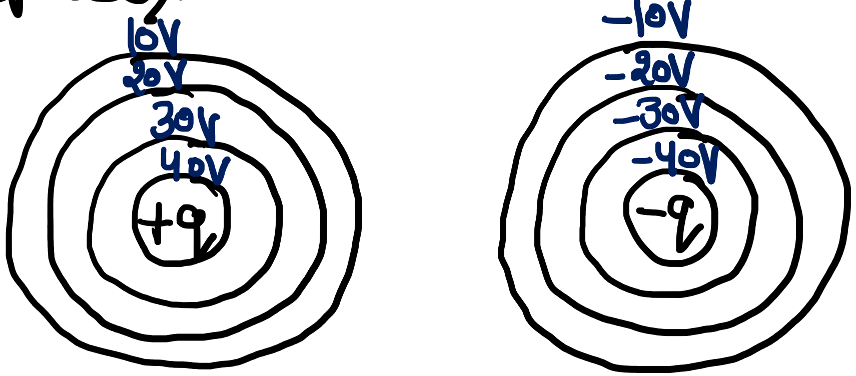
$$|\vec{E}| = \sqrt{(-1)^2 + 4^2 + (-1)^2} = \sqrt{18} = 3\sqrt{2}$$

[Equipotential Surface]

→ Locus of all points or regions that has same electric potential at every point is called an equipotential surface. (By joining points of same potential, we can draw equipotential surface).

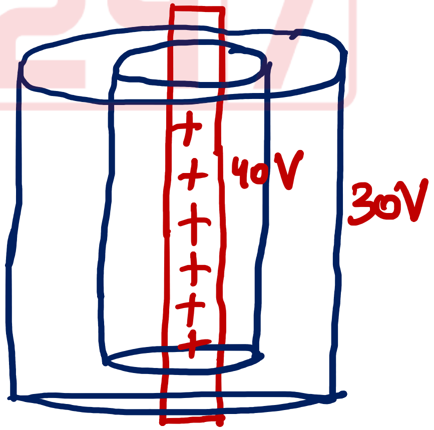
Equipotential surface for Point Charges –

infinite concentric shell with charge at centre.



Equipotential surface for Linear charge –

Coaxial hollow cylinders with linear charge as axis.



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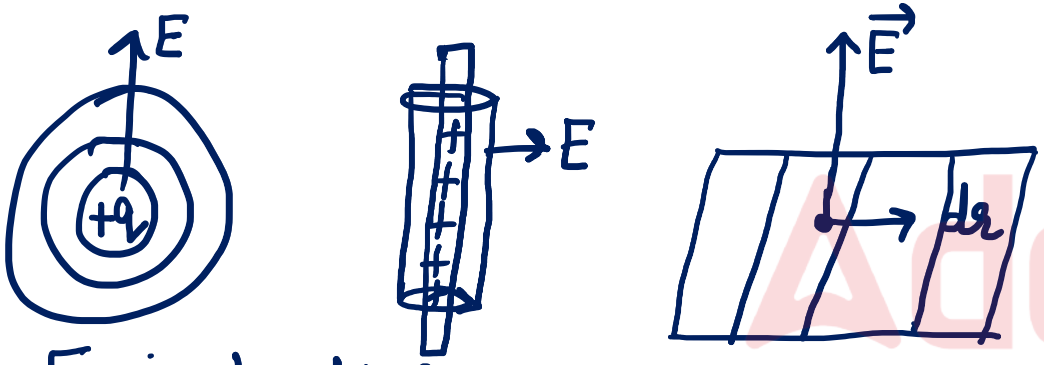
[Properties of Equipotential Surface]

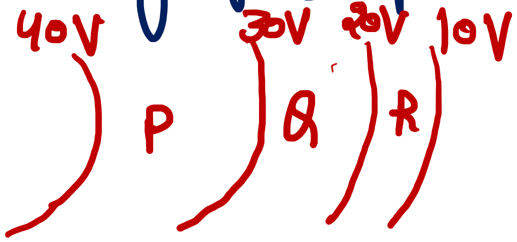
1. The potential difference between any two points on equipotential surface is zero.
2. No work is done by (or against) electric force in moving a charge over equipotential surface. ($W_{electric} = -W_{ext} = V_B - V_A = 0$)
3. Electric field ($\vec{E}$) is always perpendicular to equipotential surface at each point of the surface.



$$dV = -\vec{E}\cdot\vec{dr} = -E\,dr\cos\theta$$

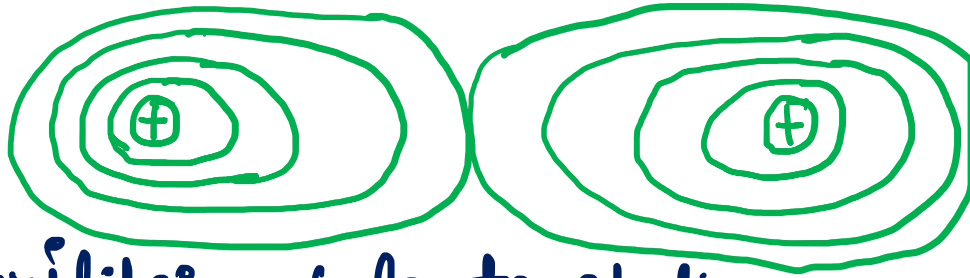
$$0 = -E\,dr\cos\theta \Rightarrow \vec{E} \perp \vec{dr}$$

4. Equipotential surfaces are crowded together in a region of strong field whereas they are relatively far apart where the field is weak.



$\Rightarrow E_P < E_Q < E_R$

→ equipotential surface for two equal positive charges kept close to each other.



→ Two equipotential surfaces can never intersect each other.

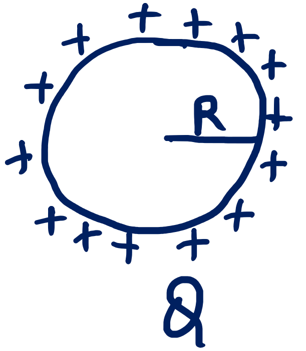
→ A charged conductor of any shape under equilibrium (electrostatic condition) is an equipotential surface. (Because $\vec{E}$ is always perpendicular to surface of conductor).

Electric Potential due to Charged Conducting Sphere (Solid/Hollow) -

Any charge given to a conductor always resides on its outer surface.

(i) Outside - At distance 'r' from centre $\Delta V = -\int \vec{E}\cdot\vec{dr}$

$r > R$, $V_P - V_\infty = -\int_\infty^P \vec{E}\cdot\vec{dr} = -\int_\infty^P E\,dr\cos\theta$



$$V_P - V_\infty = -\int_\infty^P E\,dr\cos 0^\circ = -\int_\infty^P E\,dr = -\int_\infty^P \frac{kQ}{r^2}\,dr$$ and $V_\infty = 0$

$$V_P = -kQ\int_\infty^r \frac{1}{r^2}dr = -kQ\left[-\frac{1}{r}\right]_\infty^r = kQ\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{kQ}{r}$$

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$$\boxed{V_p = \frac{kQ}{r}} \Rightarrow \text{outside } V \propto \frac{1}{r}$$

and Potential at surface

$r = R$, $V_s = \dfrac{kQ}{R}$

(ii) Inside : at distance 'r' from centre $(r<R)$

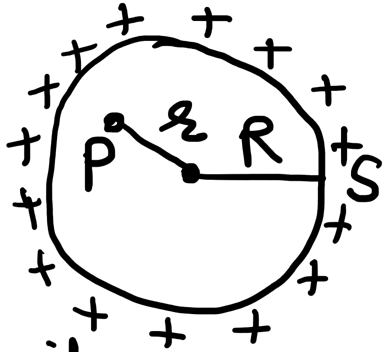
$E = 0$ inside a Conductor

$$V_p - V_s = -\int_S^P \vec{E}\cdot\vec{dr} = 0 \Rightarrow \boxed{V_p = V_s}$$

$$V_p = \frac{kQ}{R}$$

→ inside (V = constant)

↳ equipotential Volume (3-D)



$$\boxed{V_{inside} = V_{surface}}$$

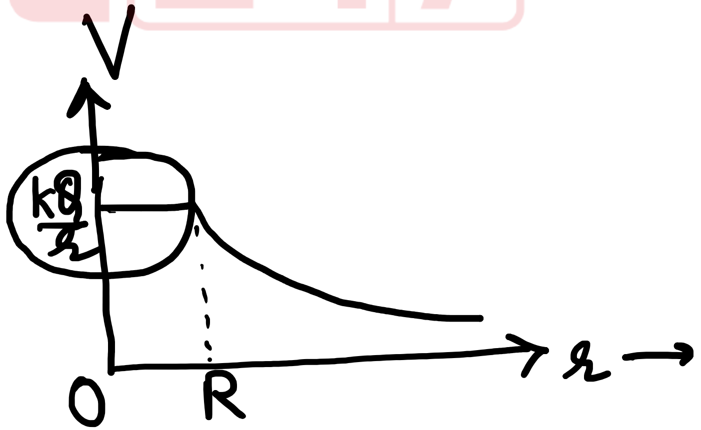
→ conducting (Hollow/ solid)

→ non-conducting (Hollow)

Graph of V v/s r –

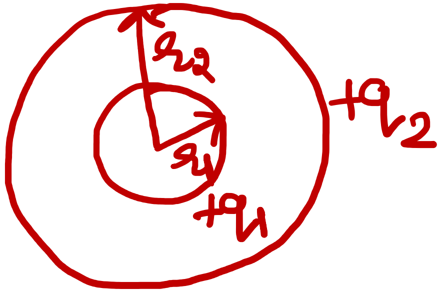
i) Outside $(r>R)$ $\quad V = \dfrac{kQ}{r} \Rightarrow V \propto \dfrac{1}{r}$

ii) Inside $\quad V = \dfrac{kQ}{R} = \text{constant}$



Q15. Figure shows two concentric conducting shells of radii r_1 and r_2 carrying uniformly distributed charges q_1 and q_2 respectively. Find potential at distance 'r' from centre, where

(i) $r < r_1$ (ii) $r_1 < r < r_2$ (iii) $r > r_2$



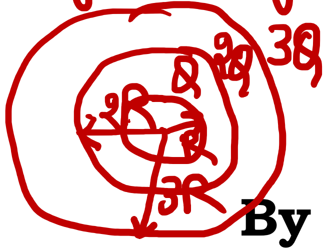
Sol. (i) $r < r_1$, $\underset{}{V_p} = \underset{\text{(inside)}}{V_1} + \underset{\text{(inside)}}{V_2} = \dfrac{kq_1}{r_1} + \dfrac{kq_2}{r_2}$

(ii) $r_1 < r < r_2$, $V_p = \underset{\text{(outside)}}{V_1} + \underset{\text{(inside)}}{V_2} = \dfrac{kq_1}{r} + \dfrac{kq_2}{r_2}$

(iii) $r > r_2$, $V_p = \underset{\text{outside}}{V_1} + \underset{\text{outside}}{V_2} = \dfrac{kq_1}{r} + \dfrac{kq_2}{r}$

Q16. Three concentric shells. Find potential at the surface of 2nd shell

a) $\dfrac{5}{2}\dfrac{kQ}{R}$ b) $\dfrac{1}{2}\dfrac{kQ}{R}$ c) $\dfrac{3}{2}\dfrac{kQ}{R}$ d) $\dfrac{7}{2}\dfrac{kQ}{R}$

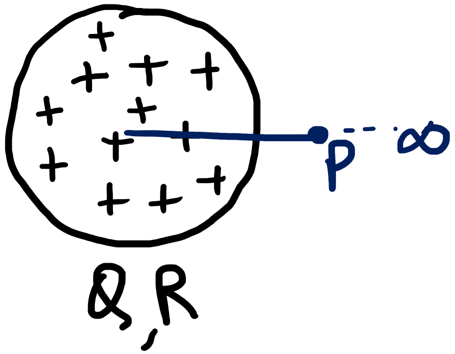


Sol. $V_{2nd\ shell\ (surface)} = V_{1\ outside} + V_{2\ surface} + V_{3\ inside} = \frac{kQ}{2R} + \frac{k(2Q)}{2R} + \frac{k(3Q)}{3R}$

$= \frac{kQ}{R}\left(\frac{1}{2}+1+1\right) = \frac{5}{2}\frac{kQ}{R}$

Electric Potential due to Non-conducting (insulating sphere → Solid) (Uniformly charged) –

Note : Charge will not come to surface, instead it, will distribute in whole volume.



(i) Outside : At distance 'r' from centre. $(r>R)$

$\infty \longrightarrow P \qquad V_P - V_\infty = -\int_\infty^P \vec{E}\cdot\vec{dr} = \int_\infty^r \frac{kQ}{r^2}\,dr = \frac{kQ}{r}$ (outside, $V \propto \frac{1}{r}$)

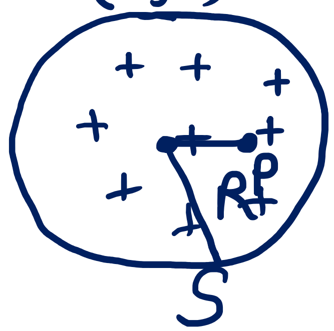
ii) At surface, $r = R$, $V_S = \frac{kQ}{R}$

iii) Inside : at distance 'r' from centre

$r < R$

$E_P = \frac{\rho r}{3\epsilon_0}$

(R, Q)



$$V_P - V_S = -\int_S^P \vec{E}\cdot\vec{dr} = -\int_S^P \frac{kQr}{R^3}dr = \frac{-kQ}{R^3}\int_R^r r\,dr$$

$$\therefore V_P - V_S = \frac{-kQ}{R^3}\left[\frac{r^2}{2}\right]_R^r = -\frac{kQ}{R^3}\left[\frac{r^2}{2} - \frac{R^2}{2}\right] \Rightarrow V_P - \frac{kQ}{R} = -\frac{kQ}{R^3}\left[\frac{r^2}{2} - \frac{R^2}{2}\right]$$

$$\Rightarrow V_P - \frac{kQ}{R} = \frac{-kQr^2}{2R^3} + \frac{kQ}{2R} = \frac{3kQ}{2R} - \frac{kQr^2}{2R^3} \Rightarrow \boxed{V_P = \frac{kQ}{R}\left[\frac{3}{2} - \frac{1}{2}\frac{r^2}{R^2}\right]}$$

→ inside non-conducting solid sphere.

→ From centre to surface ($r \uparrow \Rightarrow V_P$ - decrease)

* Potential at centre ($r=0$), $V_c = \frac{3}{2}V_S$

By – Sonu Sir

Graph of V v/s r

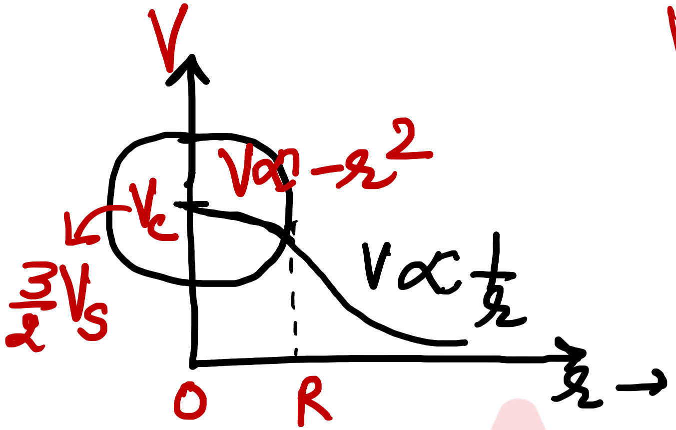
outside, $r > R$

$V = \frac{kQ}{r}$, $V \propto \frac{1}{r}$

inside : $r < R$

$V = \frac{kQ}{R}\left[\frac{3}{2} - \frac{1}{2}\frac{r^2}{R^2}\right]$

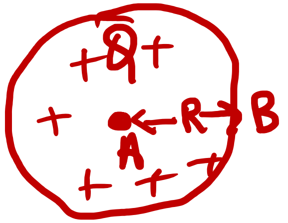
$V_{max} = \frac{3}{2} V_s$ (centre)



Q.17. Find the electric work done in bringing a charge q from A to B in a sphere of charge Q distributed uniformly throughout its volume.

a) $\frac{3}{2}\frac{kQq}{R}$ b) $-\frac{3}{2}\frac{kQq}{R}$

c) $-\frac{1}{2}\frac{kQq}{R}$ d) $\frac{1}{2}\frac{kQq}{R}$



Sol. $A \longrightarrow B$, $V_B - V_A = \frac{W_{A\to B}}{q}$

surface centre

$V_s - \frac{3}{2}V_s = \frac{W_{ext}}{2} \Rightarrow W_{ext} = -\frac{1}{2}\frac{kQq}{R}$

$W_{electric} = -W_{ext} = \frac{1}{2}\frac{kQq}{R}$

[Electric Potential Energy]

Electric potential energy is defined for system of two or more charges.
(U for single charge is zero).

Q —— r —— • P

$E_p = \frac{kQ}{r^2}$

$V_p = \frac{kQ}{r}$

$U_p = 0$

Difference in P.E. is defined

Configuration A: q_1 q_2 — initial

Configuration B: q_1 q_2 — final

$\Delta U = U_f - U_i$ = Work done by external agent to change configuration A to configuration B.

$\Delta U = U_f - U_i = W_{external}$ agent to bring that change (such that $\Delta K.E. = 0$)

Finding Potential energy of two charges at separation 'r' –

q_1 q_2 — initial

$q_1 \xleftrightarrow{r} q_2$ — Final

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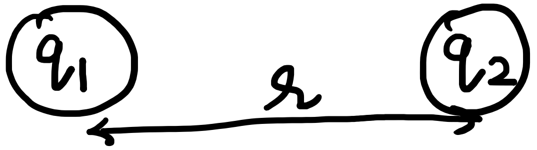
Assumption, $\left.\begin{array}{l} U_i = 0 \\ U_\infty = 0 \end{array}\right\}$ initial



$\Delta U = U_f - U_i$ = Work done by external agent in bringing each charge from ∞ to present configuration.

$= W_{ext.} = -W_{electric}$

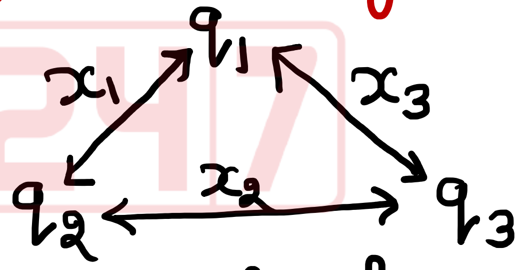
$\boxed{U = \dfrac{kq_1q_2}{r}} \rightarrow q_1$ and q_2 with sign.

Electric Potential Energy of a system of three charges –

$U_i = 0$

q_1	q_2	q_3
∞	∞	∞

initial



final

$U_f = U$

$\Delta U = U_f - U_i$ = $W_{ext.}$ in bringing q_1, q_2, q_3 from ∞ to present configuration.

$$\left[U = \frac{kq_1q_2}{x_1} + \frac{kq_1q_3}{x_3} + \frac{kq_2q_3}{x_2} \right] \rightarrow \text{system of charges.}$$

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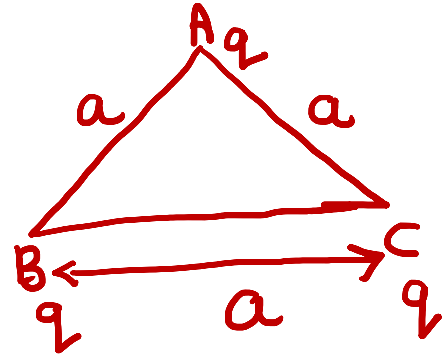
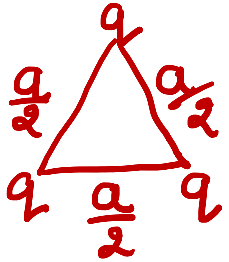
Q18. Three equal charges q are placed at the corners of an equilateral triangle of side 'a'. Find out potential energy of charge system.



Sol.

$$U = \frac{kqq}{a} + \frac{kqq}{a} + \frac{kqq}{a} = \frac{3kq^2}{a} \text{ (initial)}$$

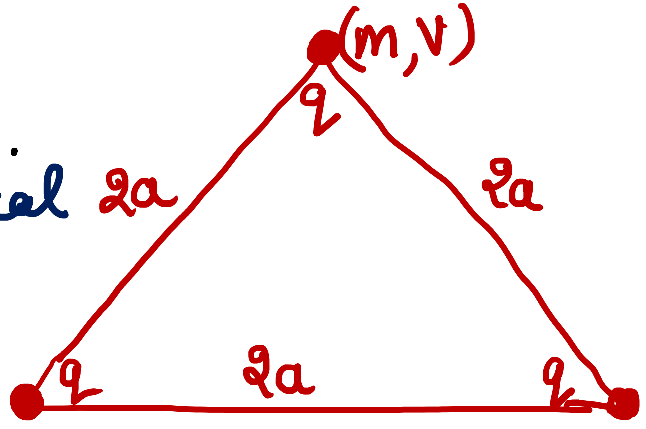
→ Calculate work required to decrease the side of triangle to $\frac{a}{2}$.



$$U_f = \frac{kqq}{\frac{a}{2}} \times 3 = \frac{6kq^2}{a}$$

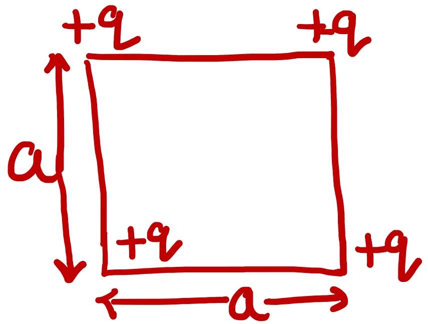
$$\Delta U = U_f - U_i = W_{ext} = \frac{6kq^2}{a} - \frac{3kq^2}{a} = \frac{3kq^2}{a}$$

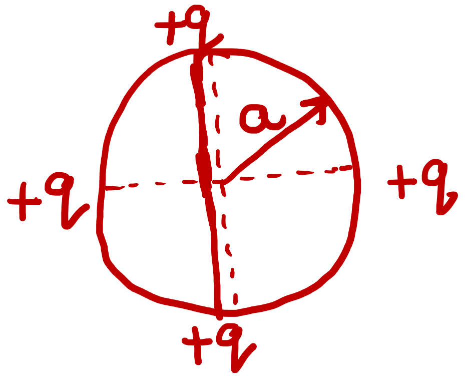
→ If the charges are released from the shown position and each of them has same mass 'm', then find the speed of each particle when they lie on triangle of side 2a.



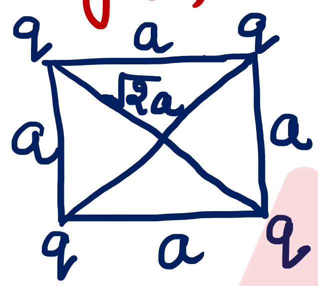
Release	Final	Conservation of mechanical energy –
$U_i = \frac{3kq^2}{a}$	$U_f = \frac{3kq^2}{2a}$	$U_i + K_i = U_f + K_f$
$K_i = 0$	$K_f = \frac{3}{2}mv^2$	$\Rightarrow v = \sqrt{\frac{kq^2}{ma}}$

Q19. Consider the configuration of a system of four charges each of equal to $+q$. Find the work done by external agent in changing the configuration of system from Fig (I) and Fig (II).

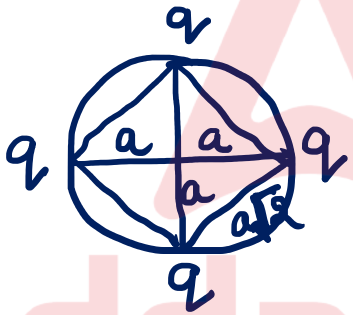




Sol. $U_{initial} = \dfrac{4kq^2}{a} + \dfrac{2kq}{\sqrt{2}a}$



$U_{final} = \dfrac{4kq^2}{\sqrt{2}a} + \dfrac{2kq}{2a}$



$W_{ext} = \Delta U = U_f - U_i$

$$= \frac{4kq^2}{\sqrt{2}a} + \frac{2kq^2}{2a} - \frac{4kq^2}{a} - \frac{2kq^2}{\sqrt{2}a} = \frac{2kq^2}{\sqrt{2}a} - \frac{3kq^2}{a} = \frac{kq^2}{a}(\sqrt{2} - \sqrt{3})$$

Q20. Two identical particles, each having a charge of 2×10^{-4} C and mass of 10 g are kept at a separation of 10 cm and then released. What would be the speeds of the particles when the separation becomes large?

a) $200\ ms^{-1}$ b) $400\ ms^{-1}$ c) $600\ ms^{-1}$ d) $800\ ms^{-1}$

By – Sonu Sir

Sol. $q^+ \xleftrightarrow{\; r \;} q^+$ $\quad q = 2\times10^{-4}\,C$, $r = 10\,cm$, $m = 10\,g$

$V \leftarrow \underset{m}{q^+} - - - - - - - \underset{m}{q^+} \rightarrow V$

$U_i = \dfrac{kq^2}{r}$

$K_i = 0$

$U_f = \dfrac{kqq}{\infty} = 0$

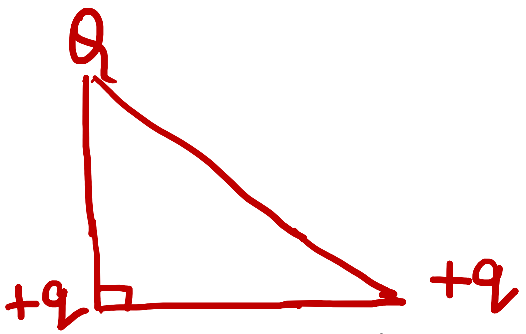
$K_f = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$

$U_i + K_i = U_f + K_f$

$\dfrac{kq^2}{r} + 0 = 0 + 2\times\frac{1}{2}mv^2 \Rightarrow \dfrac{9\times10^9\times(2\times10^{-4})^2}{10\times10^{-2}} = 10\times10^{-3}\times V^2$

$\Rightarrow V = \sqrt{9\times10^2\times10^2\times4} = 3\times2\times10^2 = 600\ m/s$

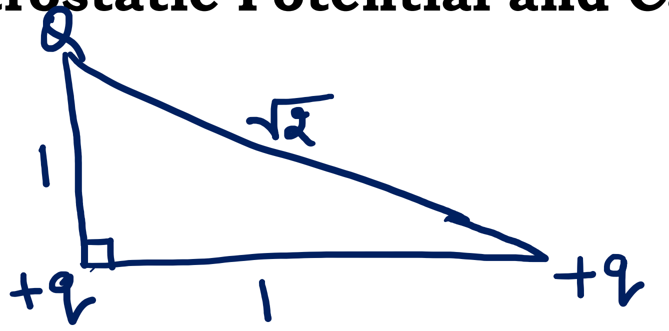
Q21. Three charge Q, +q and +q are placed at the vertices of a right angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, if the value of Q is



a) $-2q$ b) $\dfrac{-q}{1+\sqrt{2}}$ c) $+q$ d) $\dfrac{-\sqrt{2}q}{\sqrt{2}+1}$

By – Sonu Sir

Sol.



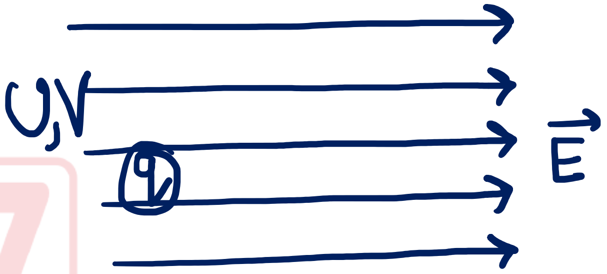
$$U = \frac{kQq}{1} + \frac{kqq}{1} + \frac{kQq}{\sqrt{2}}$$

$$0 = kq\left(Q + q + \frac{Q}{\sqrt{2}}\right) \Rightarrow Q + q + \frac{Q}{\sqrt{2}} = 0$$

$$\Rightarrow Q + \frac{Q}{\sqrt{2}} = -q \quad \text{or} \quad Q\left(1 + \frac{1}{\sqrt{2}}\right) = -q \quad \Rightarrow Q = \frac{-\sqrt{2}q}{\sqrt{2}+1}$$

Potential Energy of Single Charge in an External Field-

$$U = qV \Rightarrow W_{ext} = \Delta U = U_f - U_i$$
$$= qV$$

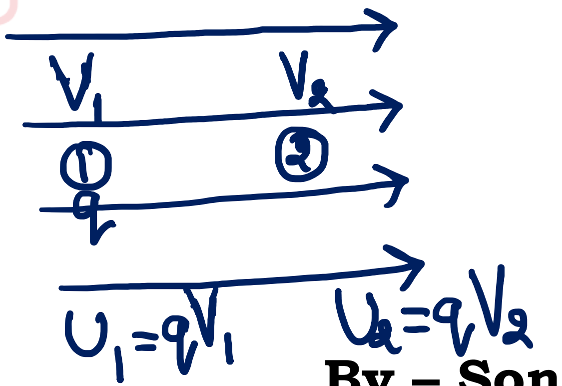


If charges move in External Electric Field-

$$\Delta U = U_2 - U_1 = qV_2 - qV_1 = q(V_2 - V_1)$$

$$\therefore \Delta U = q\Delta V$$

↳ with sign



Q22. Find the change in potential energy of an electron when it moved through a potential difference of 1V.

Sol. $\Delta V = 1V$ and $\Delta U = q\Delta V = eV$ (electron volt)

$$|\Delta U| = |q\Delta V| = |e\Delta V| = 1.6 \times 10^{-19} C \times 1V = 1.6 \times 10^{-19} J$$

Q23. Calculate the change in potential energy of a particle of charge $+q$ that is brought from a distance of $3r$ to a distance of $2r$ in the electric field of charge $-q$?

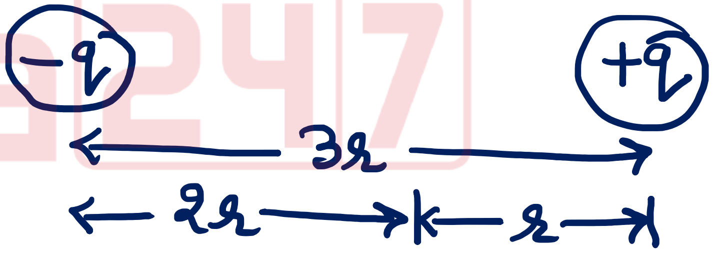
a) $\frac{kq^2}{r}$ b) $\frac{-kq}{6r}$ c) $\frac{kq^2}{4r^2}$ d) $\frac{-kq^2}{4r^2}$

Sol.

$$\Delta U = q\Delta V = +q(V_2 - V_1)$$

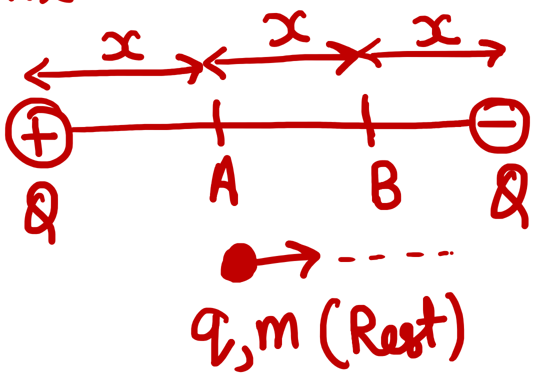
$$= q\left(\frac{-kq}{2r} - \left(-\frac{kq}{3r}\right)\right)$$

$$= q\left(\frac{-3kq + 2kq}{6r}\right) = \frac{-kq^2}{6r}$$



Q24. Two point charges Q and −Q are fixed at a distance of 3x as shown. A dust particle with mass m and charge q starts from rest at point 'A' and move in a straight line to point 'B'. What is its speed 'v' at point B.

a) $v = 2\sqrt{\frac{kqQ}{mx}}$ b) $v = \sqrt{\frac{2kqQ}{mx}}$ c) $v = \sqrt{\frac{kqQ}{mx}}$ d) $v = \sqrt{\frac{kqQ}{2mx}}$



Sol. All forces are conservative (electrostatic force)
→ Conservation of mechanical energy.

$$U_A + K_A = U_B + K_B$$

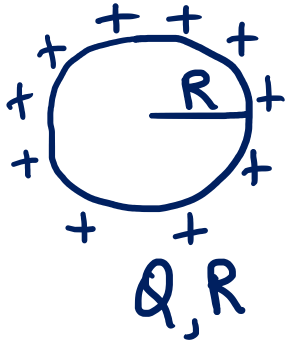
$$qV_A + 0 = qV_B + \frac{1}{2}mv^2$$

$$qV_A - qV_B = \frac{1}{2}mv^2 \Rightarrow q\left[\left(\frac{kQ}{x} - \frac{kQ}{2x}\right) - \left(\frac{-kQ}{x} + \frac{kQ}{2x}\right)\right] = \frac{1}{2}mv^2$$

$$\therefore q\frac{kQ}{x}\left[1 - \frac{1}{2} + 1 - \frac{1}{2}\right] = \frac{1}{2}mv^2 \Rightarrow \frac{kqQ}{x} = \frac{1}{2}mv^2 \Rightarrow \boxed{v = \sqrt{\frac{2kqQ}{mx}}}$$

Q25. Find the self energy of a charged spherical shell having charge Q and radius R.

Sol. Let at some time, q charge is present on shell and potential of shell is V.



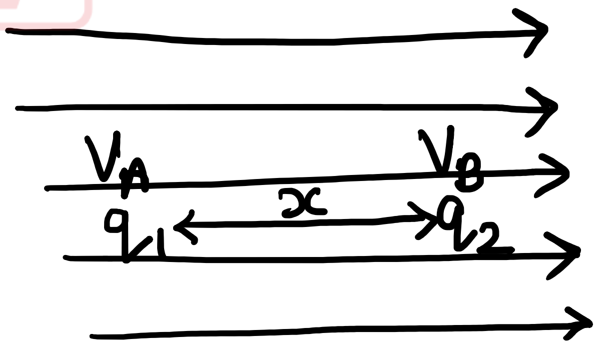
$$V - V_\infty = \frac{d}{dq}(W_{ext}) \Rightarrow \frac{kq}{R} = \frac{d}{dq}\left(\frac{kq}{R}dq\right)$$

$$\therefore dW_{ext} = \frac{kq}{R}dq \Rightarrow W_{ext} = \int_0^Q q\,dq = \frac{k}{R}\left[\frac{q^2}{2}\right]_0^Q = \frac{k}{R}\left[\frac{Q^2}{2} - \frac{0^2}{2}\right]$$

$$\therefore W_{ext} = \frac{kQ^2}{2R} \text{ (stored as P.E.)} \Rightarrow U = \frac{kQ^2}{2R}$$

Potential energy of a system of two charges in an External field.

If No external field, $U = \frac{kq_1q_2}{x}$



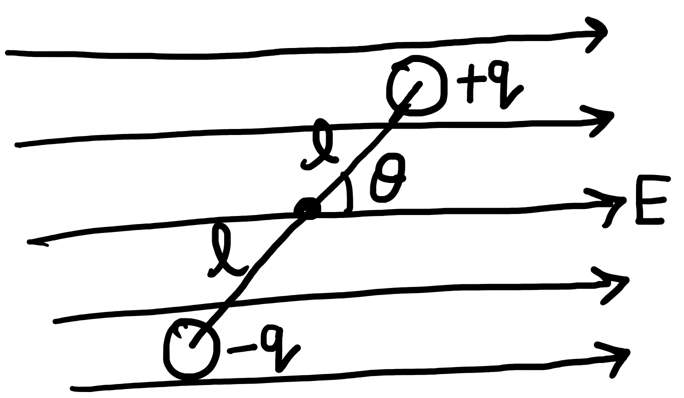
[Potential Energy of Electric Dipole In a Uniform Electric Field]

$\vec{p} = q \times 2l$ (−ve to +ve)

$\vec{F}_{net} = 0$ (always)

$\tau = \vec{p} \times \vec{E} = pE\sin\theta$

(Torque aligns the dipole in the direction of $\vec{E}$)



$\Delta U = U_f - U_i = W_{ext} = -W_{electric}$

If dipole is rotated from θ_1 to θ_2

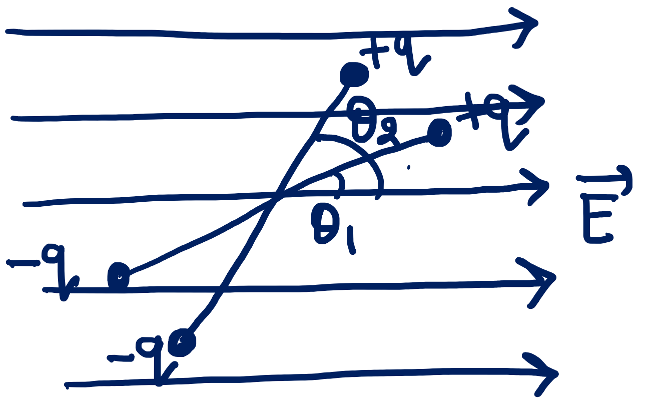
Work = τ × Angular displacement

$\tau = \vec{p} \times \vec{E} = pE\sin\theta$

Rotation $\theta_1 \longrightarrow \theta_2$ (anticlockwise)

Work done by electric field

$dW = \vec{\tau} \cdot \vec{d\theta}$

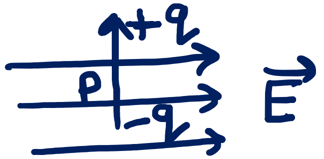


W_{net} (electric force) $= \int dW = \int \tau . d\theta = \int \tau d\theta \cos 180° = \int_{\theta_1}^{\theta_2} -\tau d\theta = -\int_{\theta_1}^{\theta_2} pE \sin\theta \, d\theta$

$= -pE \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta = -pE \left[-\cos\theta\right]_{\theta_1}^{\theta_2} = pE\left[\cos\theta\right]_{\theta_1}^{\theta_2} = pE\left[\cos\theta_2 - \cos\theta_1\right]$

$\Delta U = U_{\theta_2} - U_{\theta_1} = -W_{electric\ force} \Rightarrow U_{\theta_2} - U_{\theta_1} = -pE(\cos\theta_2 - \cos\theta_1)$

(let at $\theta = 90°$, $\vec{E} \perp \vec{p} \Rightarrow U_{90°} = 0$



$U_{\theta_2} - U_{\theta_1} = -pE(\cos\theta_2 - \cos\theta_1)$

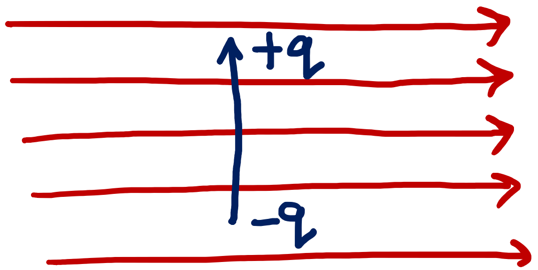
Let $\theta_1 = 90°$, $U_{\theta_2} - U_{90°} = -pE(\cos\theta_2 - \cos 90°) \Rightarrow U_{\theta_2} - 0 = -pE\cos\theta_2$

$\Rightarrow U_{\theta_2} = -pE\cos\theta_2 \Rightarrow \boxed{U_\theta = -pE\cos\theta}$ or $\boxed{U_\theta = -\vec{p} \cdot \vec{E}}$

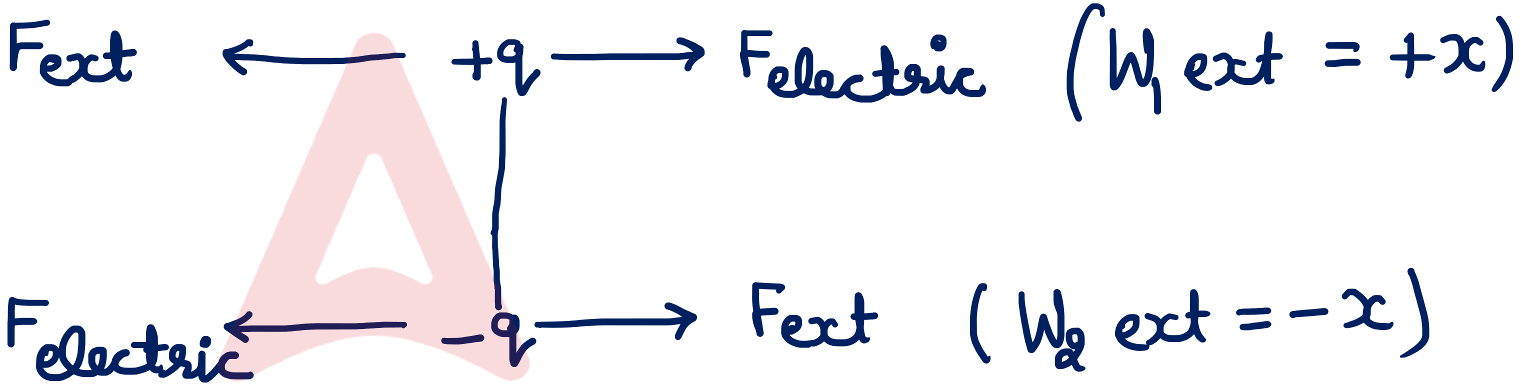
Position of Zero Energy $(\theta = 90^\circ)$

$$U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$$

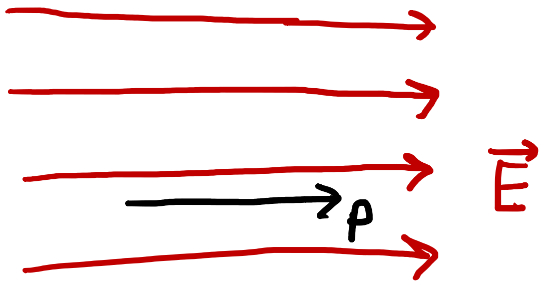
if $\theta = 90^\circ \Rightarrow U = -pE\cos 90^\circ \Rightarrow U = 0$



$$U = W_1(ext) + W_2(ext)$$
$$= +x - x = 0$$



[Stable and Unstable Equilibrium]



$\theta = 0°$

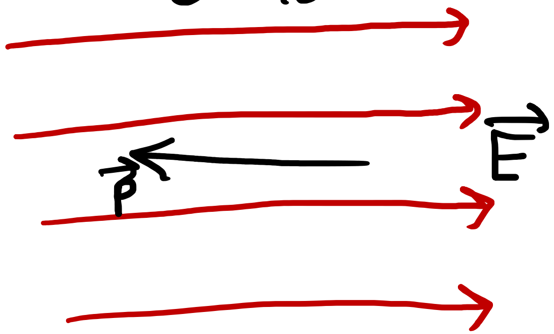
$F_{net} = 0$ (always)

$\tau = pE\sin\theta \Rightarrow \tau = 0$ (Stable Equilibrium)

$U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$

$U = -pE\cos\theta = -pE$ (min.)

$\theta = 180°$



$\vec{F}_{net} = 0$

$\tau = pE\sin\theta$

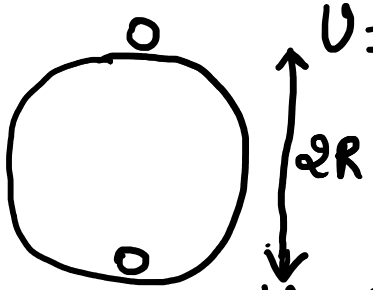
$\tau = 0$

(Unstable Equilibrium)

$U = -pE\cos\theta = -pE\cos 180°$

$= +pE$ (max.)

*



$U = mg(2R)$ (Unstable equilibrium)

$U = 0$ (Stable equilibrium)

By – Sonu Sir

Work done in Rotating a dipole from θ_1 to θ_2 →

$$W_{ext} = \Delta U = U_{\theta_2} - U_{\theta_1} = -pE\cos\theta_2 - (-pE\cos\theta_1)$$

$$= -pE\cos\theta_2 + pE\cos\theta_1 = pE[\cos\theta_1 - \cos\theta_2]$$

$$\therefore W_{ext} = pE(\cos\theta_1 - \cos\theta_2)$$

Q26. Find the amount of work done in rotating an electric dipole, of dipole moment 3.2×10^{-8} Cm, from its position of stable equilibrium, to the position of unstable equilibrium, in a uniform electric field intensity 10^4 N/C.

a) 6.4×10^{-4} J b) 3.2×10^{-4} J c) 4.8×10^{-4} J d) 12.8×10^{-4} J

Sol. initial ⟶ final

(stable equilibrium) (unstable equilibrium)

$\theta = 0°$ $\theta = 180°$

$$W_{ext} = \Delta U = U_{180°} - U_{0°} = -pE\cos 180° - (-pE\cos 0°) = -pE(-1) + pE = 2pE$$

$$\therefore W_{ext} = 2 \times 3.2 \times 10^{-8} \times 10^4 = 6.4 \times 10^{-4} \text{ J}$$

Q27. An electric dipole consists of two opposite charges each of magnitude $1\mu C$, separated by 2cm. The dipole is placed in an external uniform field of $10^5 NC^{-1}$ intensity. Find the

a) Maximum torque exerted by the field on the dipole and

b) work done in rotating the dipole through 180° starting from the position $\theta = 0°$.

Sol. a)

$$\vec{E} = 10^5 \ N/C$$

$$\vec{p} = q \times 2l = 1\times10^{-6} \times 2\times10^{-2}$$

$$= 2\times10^{-8} \ Cm$$

$$\tau = \vec{p} \times \vec{E} = pE\sin\theta$$

$$\tau_{max} \ (\theta = 90°) = pE$$

$$= 2\times10^{-8} \times 10^5$$

$$= 2\times10^{-3} \ Nm$$

b) $$W_{ext} = \Delta U = U_{\theta_2} - U_{\theta_1} = U_{180°} - U_{0°} = 2pE$$

$$= 2\times(2\times10^{-3}) = 4\times10^{-3} \ J$$

Q28. An electric dipole of length 4cm, when placed with its axis making an angle of 60° with a uniform electric field experiences a torque of $4\sqrt{3}$ Nm. Calculate the potential energy of the dipole, if dipole has charge of ±8nC.

a) 4J b) −4J c) 8J d) −8J

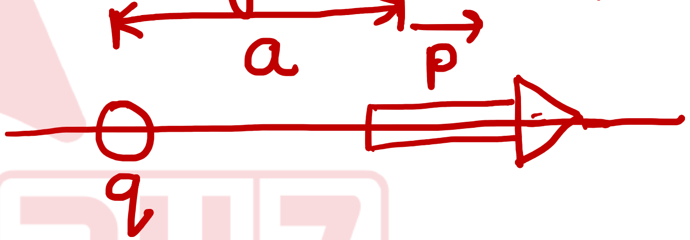
Sol. $\tau = pE\sin\theta \quad \text{—} \quad (1)$

$U = -pE\cos\theta \quad \text{—} \quad (2)$

$$\frac{\tau}{U} = \frac{pE\sin\theta}{-pE\cos\theta} = -\tan\theta \Rightarrow \frac{\tau}{U} = -\tan\theta = -\tan 60^\circ = -\sqrt{3}$$

$$\Rightarrow U = \frac{4\sqrt{3}}{-\sqrt{3}} = -4J$$

Q29. What is the potential energy of the charge and dipole system shown in figure.

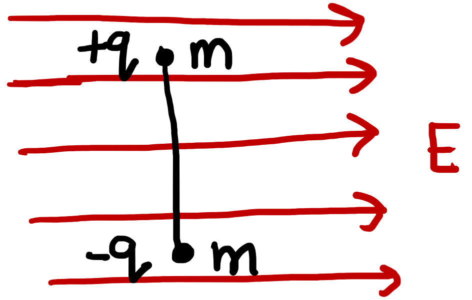


Sol.

$\vec{E} = \frac{kq}{a^2}$ $(a >>> 2l \rightarrow \text{short dipole})$

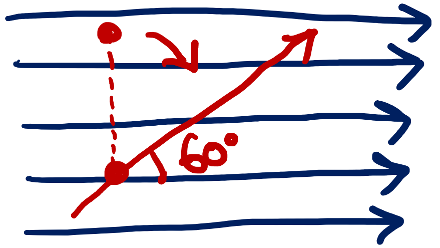
$$U = -pE\cos\theta = -p\frac{kq}{a^2}\cos 0^\circ = \frac{-kpq}{a^2} = -\frac{pq}{4\pi\epsilon_0 a^2}$$

Q30. An electric dipole is held perpendicular to a uniform Electric field as shown. If the dipole is released & free to rotate about it's centre, find the angular velocity of dipole when it is at 60° inclination with Electric field. (mass of each charge = m).



a) $\sqrt{\frac{2qE}{ml}}$ b) $\sqrt{\frac{qE}{2ml}}$ c) $\sqrt{\frac{qE}{ml}}$ d) 0

Sol.



initial (at rest)

$U_i = -pE\cos\theta = -pE\cos 90°$
$= 0$

$K_i = 0$ (Rest)

final

$U_f = -pE\cos\theta$

$K.E_f = \frac{1}{2} I\omega^2$ (where $I = ml^2 + ml^2 = 2ml^2$)

Conservation of mechanical energy, $U_i + K_i = U_f + K_f \Rightarrow 0 + 0 = -pE\cos\theta + \frac{1}{2}I\omega^2$

$\Rightarrow pE\cos\theta = \frac{1}{2}I\omega^2 \Rightarrow 2qE\cos\theta = ml^2\omega \Rightarrow \omega^2 = \frac{2qE\cos\theta}{ml} \Rightarrow \omega = \sqrt{\frac{2qE\cos\theta}{ml}}$

$\Rightarrow \omega = \sqrt{\frac{2qE\cos 60°}{ml}} = \sqrt{\frac{qE}{ml}}$

By – Sonu Sir

[Capacitor]

Capacitor (sometimes known as condensers) are used to store electrical energy and supply it at once when required.

→ Capacitors are widely used in televisions, radios, mobiles.

[battery → stored electric energy but release energy very slowly].

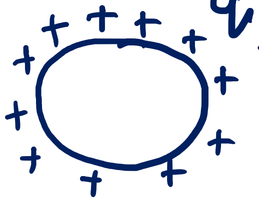
→ In capacitor, electric energy is stored by storing electric charge.

→ Capacitors are also used in –
i) Filtering a signal
ii) Tuning a radio
iii) Flicking channels on your T.V. etc.

→ Capacitors are basically conductors. The property of conductor to hold charge is called capacitance. (Capacitance is a property of conductor)

Capacitance : Ability of a conductor to hold | store electric charge or electric energy.

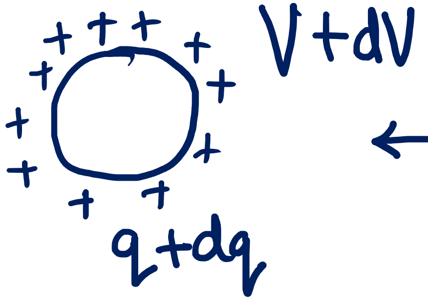
$$\boxed{C = \frac{Q}{V}}$$



q, V ← dq (repel)

work done to store dq on conductor,

$dW = dq\,V$



$\leftarrow dq$
(more repulsion)

$dW' = dq(V+dV)$

$\rightarrow$ more the value of V, it is difficult to store charge (more work required).

Capacitance depends on –
a) Shape and size of conductor
b) Properties of surrounding medium
c) Property of any dielectric/insulator introduced.

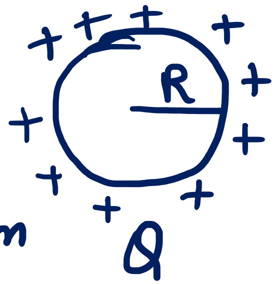
$Q \propto V$
$\Rightarrow Q = CV$
C $\downarrow$ constant

* Capacitance doesnot depend on Q and V.

* Capacitance – $\rightarrow$ unit – Farad
$\rightarrow$ dimension – $[M^{-1}L^{-2}T^{4}A^{2}]$

Capacitance of a spherical conductor or capacitor –

$$C = \frac{Q}{V} = \frac{Q}{\frac{KQ}{R}} = 4\pi\epsilon_0 R$$



$\rightarrow$ Radius of a spherical conductor is 9×10^{9} m whose capacitance is 1 Farad.

Q31. An isolated sphere has a capacitance 50 pF.

i) Calculate its radius

ii) How much charge should be placed on it to raise its potential to 10^4 V?

Sol. i) $C = 4\pi\epsilon_0 R \Rightarrow 50\times10^{-12} = 4\pi\epsilon_0 R \Rightarrow R = 9\times10^9 \times 50\times10^{-12} = 0.45\,m$

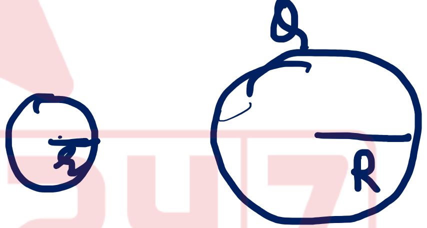
ii) $Q = CV = 50\times10^{-12}\times10^4 = 50\times10^{-8}C = 0.5\,\mu C$

Q32. Twenty seven spherical drops of radius 3 mm and carrying 10^{-12}C of charge are combined to form a single drop. Find the capacitance and the potential of the bigger drop.

Sol.

Small drop $\longrightarrow$ large drop

(Volume, charge - conserved)



$C = 4\pi\epsilon_0 R$

$27q = 1Q$

$Q = 27\times10^{-12}C$

$C = 4\pi\epsilon_0 r = \frac{1}{9\times10^9}\times9\times10^{-3} = 10^{-12}F$

$V = \frac{Q}{C} = \frac{27\times10^{-12}}{10^{-12}} = 27\,V$

Volume of 27 drops = Volume of 1 bigger drop

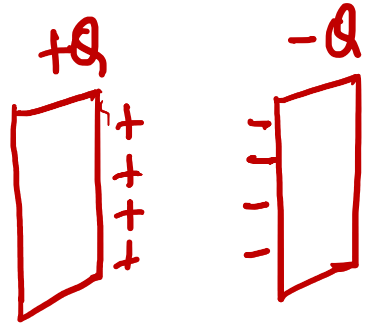
$27\times\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$\Rightarrow R = 3r = 3\times3\times10^{-3} = 9\times10^{-3}m.$

By – Sonu Sir

Energy stored in capacitor, $U = \frac{Q^2}{2C} = \frac{1}{2}QV = \frac{1}{2}CV^2$ or $U = \frac{kQ^2}{2R}$

[Parallel Plate Capacitor]



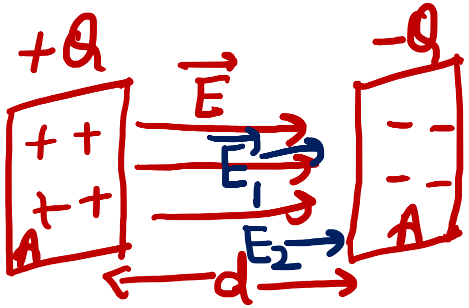
→ Electric field (electric energy) is bounded.

→ $C = \frac{Q}{V}$ [Potential (V) – decrease due to –ve plate
⇒ Capacitance – increase]

Conditions for parallel plate capacitor –

1. Area of both plate must be same (shape may be different or same)
2. Distance between plates must be small.

Capacitance of Parallel Plate Capacitor –



$$\vec{E} = \vec{E_1} + \vec{E_2} = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

$$|\Delta V| = \vec{E}\cdot\vec{d} = Ed\cos\theta = Ed = \Delta V = \frac{Qd}{\epsilon_0 A}$$

$E_1 = E_2 = \frac{\sigma}{2\epsilon_0}$ (plane sheet)

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Qd}{\epsilon_0 A}} = \frac{\epsilon_0 A}{d} \quad \text{or} \quad \boxed{C = \frac{Q}{V_+ - V_-}}$$

By – Sonu Sir

Factors on which the capacitance of a parallel plate capacitor depends –

a) Area of Plates ($C \propto A$)
b) Distance between plates ($C \propto \frac{1}{d}$)
} shape & size of conductor

c) Permittivity of medium between the plates ($C \propto \epsilon$) } medium

Q. 33. A parallel plate air capacitor consists of two circular plates of diameter 8 cm. At what distance should the plates be held so as to have the same capacitance as that of a sphere of diameter 20 cm.

a) 2mm b) 4mm c) 6mm d) 8mm

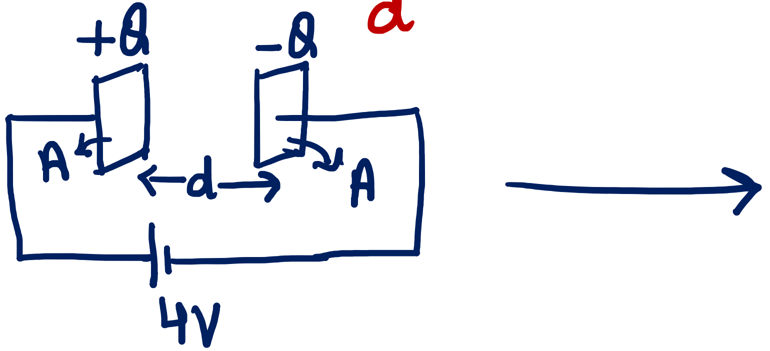
Sol. $C_1 = \frac{\epsilon_0 A}{d}$ and $C_2 = 4\pi\epsilon_0 R$ $\Rightarrow \frac{\epsilon_0 A}{d} = 4\pi\epsilon_0 R \Rightarrow \frac{A}{d} = 4\pi R$

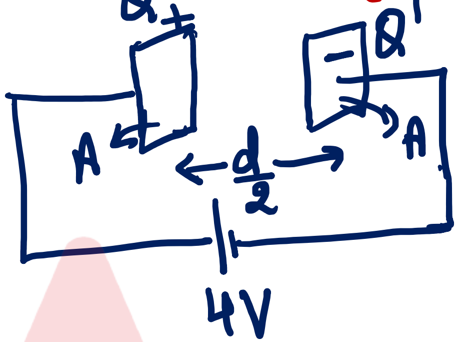
$\therefore \frac{\pi r^2}{d} = 4\pi R \Rightarrow \frac{4\times10^{-2} \times 4\times10^{-2}}{d} = 4\times10\times10^{-2} \Rightarrow d = 4\times10^{-3} m.$

Q34. A parallel plate capacitor has a plate area 'A' and distance between two plates 'd'. The capacitor is connected to 4V battery. If the separation between the plates is reduced to half, (battery remains connected), what extra charge is given by battery to positive plates?

a) $\frac{\epsilon_0 A}{d}$ b) $\frac{2\epsilon_0 A}{d}$ c) $\frac{4\epsilon_0 A}{d}$ d) $\frac{\epsilon_0 A}{2d}$

Sol.





$C' = \frac{\epsilon_0 A}{d/2} = \frac{2\epsilon_0 A}{d}$

$Q = CV = \frac{\epsilon_0 A}{d}(4) = \frac{4\epsilon_0 A}{d}$

$Q' = C'V = \left(\frac{2\epsilon_0 A}{d}\right)4 = \frac{8\epsilon_0 A}{d}$

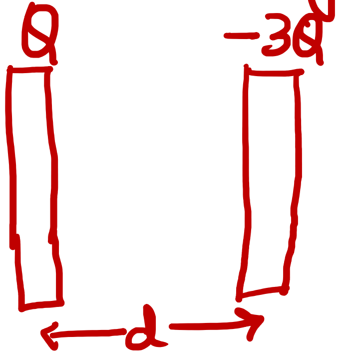
Extra charge, $Q' - Q = \frac{8\epsilon_0 A}{d} - \frac{4\epsilon_0 A}{d} = \frac{4\epsilon_0 A}{d}$

Q.35. A parallel plate capacitor has capacitance C. If the charges of the plates are Q and −3Q, Find the –

a) Charges at the inner surfaces of the plates.

b) Potential difference between the plates.

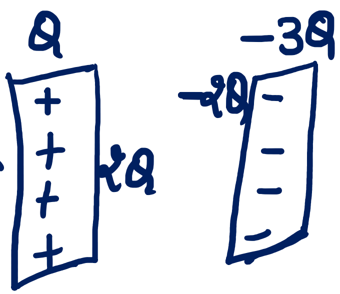
c) Charge flown if the plates are connected.



By – Sonu Sir

Sol.

$-Q = \frac{Q_1+Q_2}{2}$



$\frac{Q_1+Q_2}{2} = \frac{Q-3Q}{2} = -Q$

a) Charges at inner surfaces, $+2Q$, $-2Q$

b) potential difference, $V = \frac{2Q}{C}$

c) Charge flown if the plates are connected is $2Q$ or $-2Q$.

Q36. Two parallel plates, separated by 2mm of air have a capacitance of 3×10^{-14} F and are charged to a potential of 200V. Then without touching the plates, they are moved apart till the separation is 6mm. Find the work required to do so.

a) 5.4×10^{-10} J b) 6×10^{-10} J c) 12×10^{-10} J d) 9.8×10^{-10} J

Sol.

$Q = CV = 3\times10^{-14}\times200 = 6\times10^{-12}$ C $\Rightarrow U_i = \frac{Q^2}{2C}$

$C' = \frac{C}{3} \Rightarrow U_f = \frac{Q^2}{2C'}$

$W_{ext} = \Delta U = \frac{Q^2}{2C'} - \frac{Q^2}{2C} = \frac{3Q^2}{2C} - \frac{Q^2}{2C} = \frac{2Q^2}{2C} = \frac{Q^2}{C} = \frac{(6\times10^{-12})^2}{3\times10^{-14}} = 12\times10^{-14}$ J

Q37. A capacitor with capacitance $5\mu F$ is charged to $5\mu C$. If the plates are pulled apart to reduce the capacitance to $2\mu F$, how much work is done?

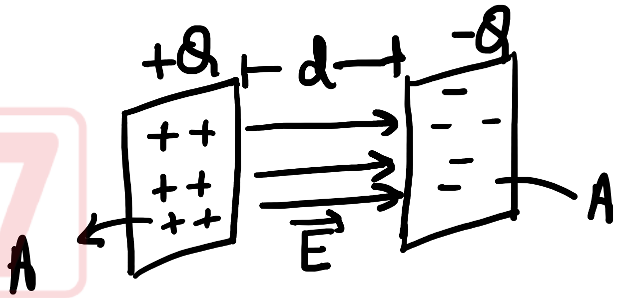
a) 6.25×10^{-6} J b) 2.16×10^{-6} J c) 2.55×10^{-6} J d) 3.75×10^{-6} J

Sol. $U_i = \frac{Q^2}{2C}$ and $U_f = \frac{Q^2}{2C'}$ $\Rightarrow W_{ext} = \frac{Q^2}{2}\left(\frac{1}{C'} - \frac{1}{C}\right) = \frac{Q^2}{2}\left(\frac{1}{2\times10^{-6}} - \frac{1}{5\times10^{-6}}\right)$

$= \frac{(5\times10^{-6})^2}{2\times10^{-6}}\left(\frac{3}{10}\right) = 3.75\times10^{-6}$ J

[Energy Density of an Electric Field]

Energy density - Energy stored per unit volume of capacitor.



$\vec{E} = \frac{\sigma}{\epsilon_0} \Rightarrow \frac{Q}{A} = \epsilon_0 E \Rightarrow Q = \epsilon_0 A \vec{E}$

Energy stored $= \frac{Q^2}{2C} = \frac{\epsilon_0 A E}{2\left(\frac{\epsilon_0 A}{d}\right)} = \frac{1}{2}\epsilon_0 E^2 Ad$

$\frac{\text{Energy}}{\text{Volume}} = \frac{U}{Ad} = \frac{\left(\frac{1}{2}\epsilon_0 E^2\right)Ad}{Ad} = \frac{1}{2}\epsilon_0 E^2 \Rightarrow$ energy density $(u) = \frac{U}{V} = \frac{1}{2}\epsilon_0 E^2$

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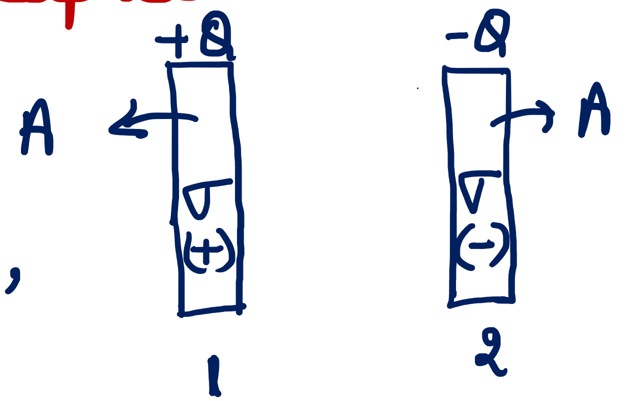
Force between the Plates of a Parallel Plate Capacitor –

* $\vec{E}$ (electric field) due to 1 at location of 2, $E = \frac{\sigma}{2\epsilon_0}$



* Force on 2 due to Electric field of 1,

$$\vec{F} = qE = QE = Q\left(\frac{\sigma}{2\epsilon_0}\right)$$

$$\therefore \boxed{F = \frac{Q^2}{2\epsilon_0 A}}$$

[Spherical Capacitor]

→ A spherical capacitor consists of two concentric spherical conducting shells.

→ The outer shell is Earthed and the inner shell is given a charge '+Q'.

$$V_2 = V_{+Q} + V_q \Rightarrow 0 = \frac{kQ}{b} + \frac{kq}{b} \Rightarrow 0 = Q + q \Rightarrow q = -Q$$

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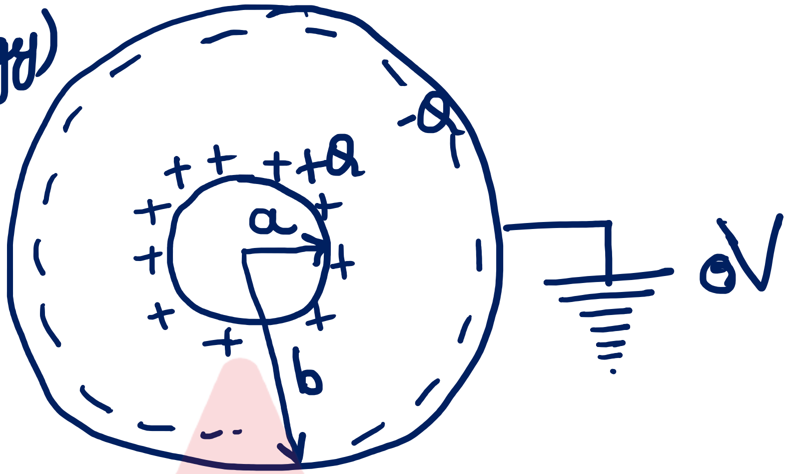
Electric field (electric energy) is bounded.



$$C = \frac{Q}{\Delta V} = \frac{Q}{V_1 - V_2}$$

$$V_1 = V_{+Q} + V_{-Q} = \frac{KQ}{a} - \frac{KQ}{b}$$

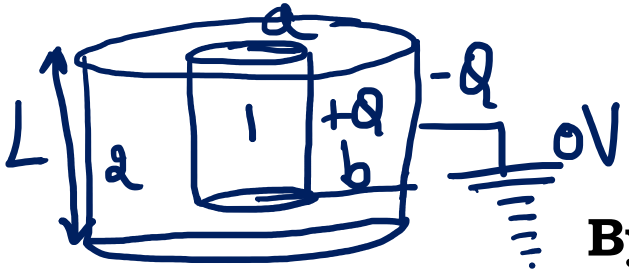
$$\Rightarrow \quad C = \frac{Q}{KQ\left(\frac{1}{a} - \frac{1}{b}\right)} \Rightarrow \boxed{C = \frac{4\pi\epsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{4\pi\epsilon_0 ab}{b-a}}$$

[Cylindrical Capacitor]

→ A cylindrical capacitor consists of two coaxial long cylinders.

→ The outer one is earthed and the inner one is given a charge +Q.

$$C = \frac{Q}{\Delta V} = \frac{Q}{V_1 - V_2}$$



Assuming Gaussian surface,

$$V_2 - V_1 = -\int_1^2 E.dr \quad .$$ We have to find $\vec{E}$.

$$\phi = \oint E.dA = \frac{Q_{inside}}{\epsilon_0} \Rightarrow E \times 2\pi r L = \frac{Q}{\epsilon_0} \Rightarrow \boxed{E = \frac{Q}{2\pi r L \epsilon_0}}$$

variable (radially outwards)

$$V_2 - V_1 = -\int_1^2 E.dr = -\int E\,dr\cos\theta = -\int_1^2 E\,dr$$

$$= -\int_a^b \frac{Q}{2\pi r L\epsilon_0} dr = \frac{-Q}{2\pi L\epsilon_0}\int_a^b \frac{dr}{r} = \frac{-Q}{2\pi L\epsilon_0}\left(\log_e r\right)_a^b$$

$$\therefore V_2 - V_1 = \frac{-Q}{2\pi L\epsilon_0}\log_e\left(\frac{b}{a}\right) \Rightarrow 0 - V_1 = \frac{-Q}{2\pi L\epsilon_0}\left[\log_e\left(\frac{b}{a}\right)\right] \Rightarrow V = \frac{Q}{2\pi L\epsilon_0}\left(\log_e \frac{b}{a}\right)$$

$$\therefore C = \frac{Q}{\frac{Q}{2\pi L\epsilon_0\left(\log_e \frac{b}{a}\right)}} = \frac{2\pi\epsilon_0 L}{\log_e\left(\frac{b}{a}\right)} \Rightarrow \boxed{C = \frac{2\pi\epsilon_0 L}{\log_e\left(\frac{b}{a}\right)}}$$

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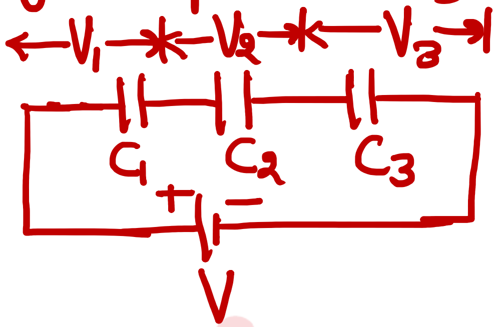
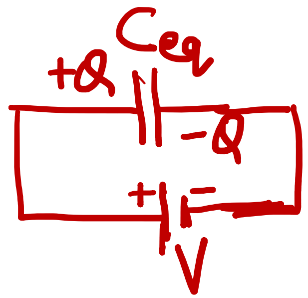
[Series Combination of Capacitors]



$\Rightarrow$



$V = V_1 + V_2 + V_3$

a) Q is same for all capacitor

b) $V = V_1 + V_2 + V_3$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

c) If 'n' Capacitors of equal capacitance C are connected in series.

$$\boxed{C_{eq} = \frac{C}{n}}$$

d) Equivalent Capacitance is always less than any of the individual capacitance.

e) Charge $Q = C_{eq} V$ on each capacitor

f) Potential, $V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$ - - - - -

[Parallel Combination of Capacitors]

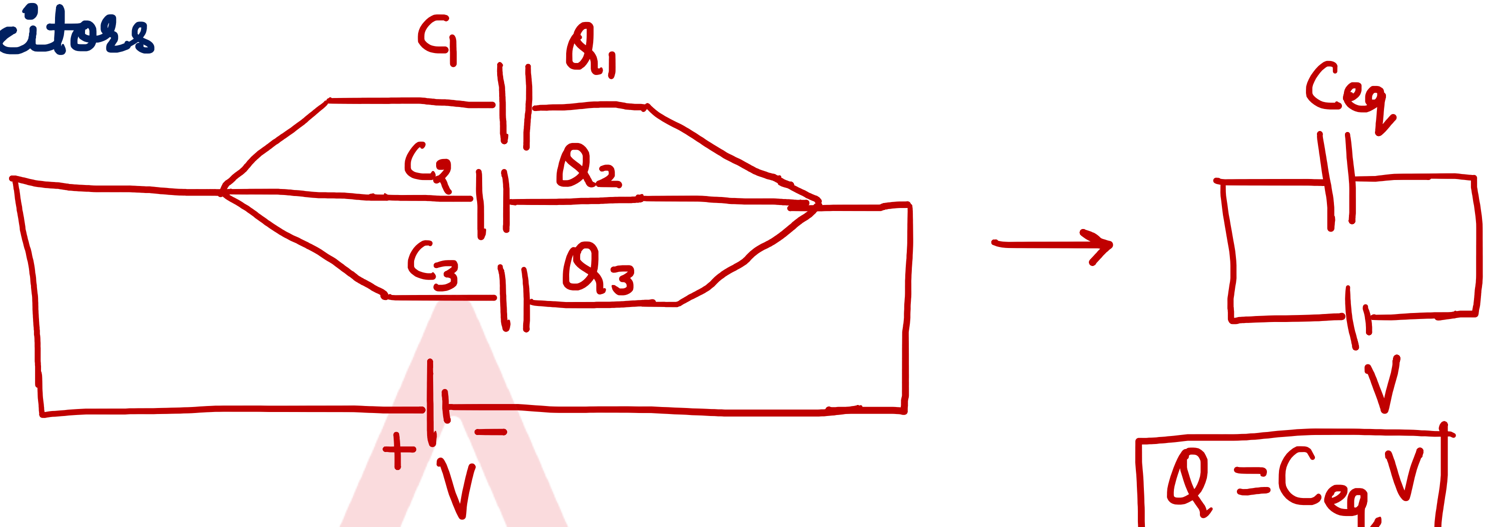
i) V is same for all capacitors

ii) Q is distributed,

$$Q = Q_1 + Q_2 + Q_3$$

$$C_{eq} V = C_1 V + C_2 V + C_3 V$$

$$\Rightarrow C_{eq} = C_1 + C_2 + C_3$$



$$\boxed{Q = C_{eq} V}$$

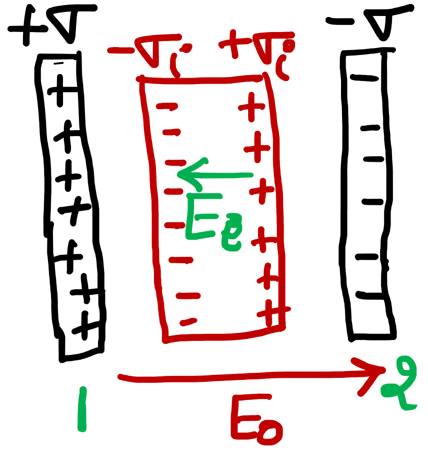
iii) If 'n' capacitors of equal capacitance C are connected in parallel

$$\boxed{C_{eq} = nC}$$

Dielectric slab between plates of Capacitor −

→ due to external electric field ($\vec{E_0}$) of plates, charge separation or shifting occurs in dielectric (Polarisation)

→ Induced charges are developed at ends of dielectric.



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→ An induced electric field ($\vec{E_i}$) is generated opposite to original Electric field ($\vec{E_0}$) –

$$\left[\begin{array}{ll} E_i < E_0 & \rightarrow \text{dielectrics} \\ E_i = E_0 & \rightarrow \text{Metals} \end{array}\right]$$

→ The resultant electric field between plates decreases.

$$E_{net} = E_0 - E_i \quad (\text{from 1 to 2})$$

Dielectric Constant : (K or ϵ_r) –

on placing a dielectric slab in an Electric field, the net $\vec{E}$ field is decreased (always).

$$E_{net} = E_0 - E_i = \frac{E_0}{K}$$

E_0 → electric field in vacuum

→ $K > 1$ for any material.

→ dielectric constant tells us how many times E_0 decrease.

Calculation for induced charge density (σ_i) –

$$E_{net} = E_0 - E_i \Rightarrow \frac{E_0}{K} = E_0 - E_i \Rightarrow E_i = E_0\left(1 - \frac{1}{K}\right)$$

$$E_0 = \frac{\sigma}{\epsilon_0} \text{ and } E_i = \frac{\sigma_i}{\epsilon_0} \Rightarrow \frac{\sigma_i}{\epsilon_0} = \frac{\sigma}{\epsilon_0}\left(1 - \frac{1}{k}\right) \Rightarrow \boxed{\sigma_i = \sigma\left(1 - \frac{1}{k}\right)}$$

$$\text{or } \frac{Q_i}{A} = \frac{Q}{A}\left(1 - \frac{1}{k}\right) \Rightarrow \boxed{Q_i = Q\left(1 - \frac{1}{k}\right)}$$

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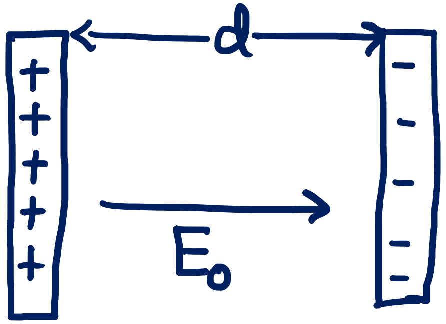
Potential Difference between Plates of Capacitor

Without Dielectric



$$V_0 = \vec{E_0} \cdot \vec{d} = E_0 d$$

With Dielectric

$$V' = \vec{E} \cdot \vec{d} = Ed$$

$$= \frac{E_0}{K} d \Rightarrow \boxed{V' = \frac{V_0}{K}}$$

$$E_{net} = E_0 - E_i = \frac{E_0}{K} = \frac{\sigma}{K\epsilon_0}$$

Capacitance of parallel plate capacitor with dielectric –

$$C = \frac{Q}{\Delta V} \Rightarrow \Delta V = \vec{E} \cdot \vec{d} = \frac{E_0}{K} d \Rightarrow \Delta V = \frac{\sigma}{\epsilon_0 K} d = \frac{Q}{A\epsilon_0 K} d$$

$$\Rightarrow \Delta V = \frac{Qd}{A\epsilon_0 K} \Rightarrow \frac{\epsilon_0 A K}{d} = \frac{Q}{\Delta V}$$

No dielectric, Capacitance in vacuum – $C_0 = \frac{\epsilon_0 A}{d}$

→ With dielectric, $C = \left(\frac{\epsilon_0 A}{d}\right) K \Rightarrow C = KC_0$ (Capacitance increases K times)

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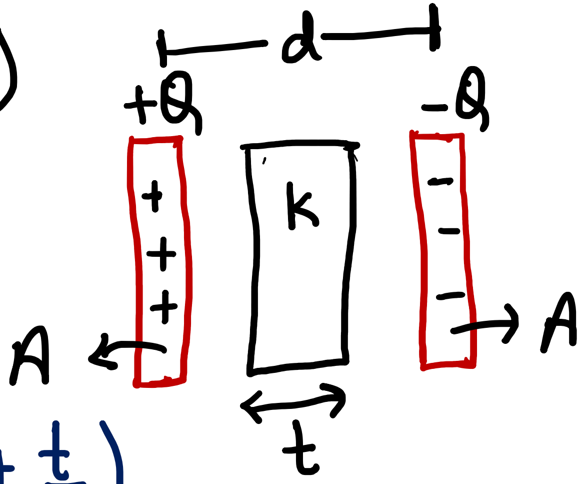
→ If a 'Metal' or 'Conductor' is used in place of Dielectric Slab –

a) $\sigma_i = \sigma$

b) $E_i = E_0$

c) $E_{net} = E_0 - E_i = 0$ or $\frac{E_0}{k} = 0$ ($\because k = \infty$)



When dielectric is partially filled:

$C = \frac{Q}{\Delta V}$, $E_0 = \frac{\sigma}{\epsilon_0}$ and $\sigma = \frac{Q}{A}$

$\Delta V = \vec{E} \cdot \vec{d} = \vec{E}_{out}(d-t) + \vec{E}_{in}(t) = E_0\left(d - t + \frac{t}{k}\right)$

$= \frac{\sigma}{\epsilon_0}\left(d - t + \frac{t}{k}\right) = \frac{Q}{A\epsilon_0}\left(d - t + \frac{t}{k}\right) \Rightarrow \frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d - t + \frac{t}{k}}$

$C = \frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d - t + \frac{t}{k}} \Rightarrow \boxed{C = \frac{\epsilon_0 A}{d - t + \frac{t}{k}}}$

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